# Velocity Logarithmic Profiles for Flows over Steep Hills

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# Plan of Presentation

- Characterization of turbulence
  - Examples, visualizations.
  - Historical perspective. Great achievements.
- Governing equations
  - RANS
  - Modelling strategies
  - Asymptotic structure of the boundary layer
  - Law of the wall formulations
- Experiments
  - Hot-wire results
  - Laser Doppler results
- Numerical simulations
  - Two-equation differential models
  - RSM
- Final remarks

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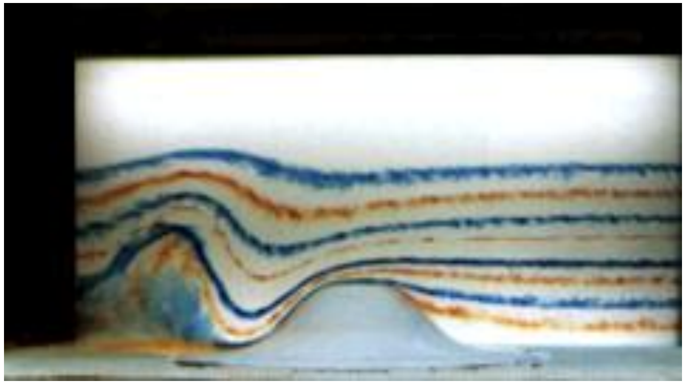
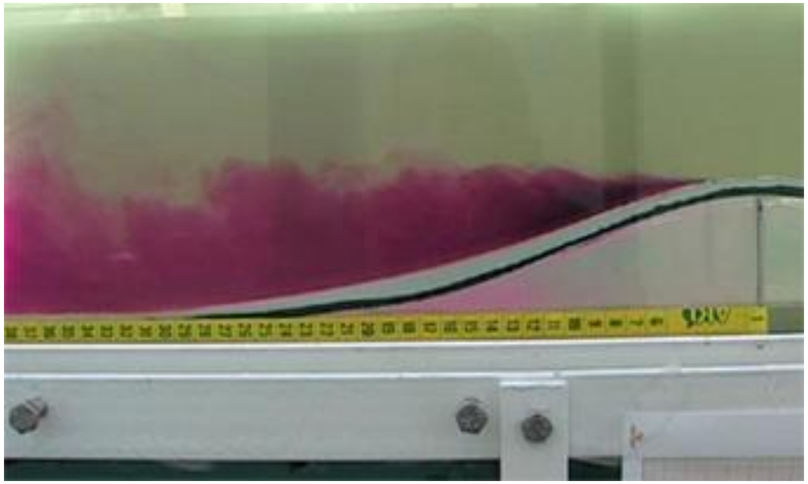
# Flow Phenomenon

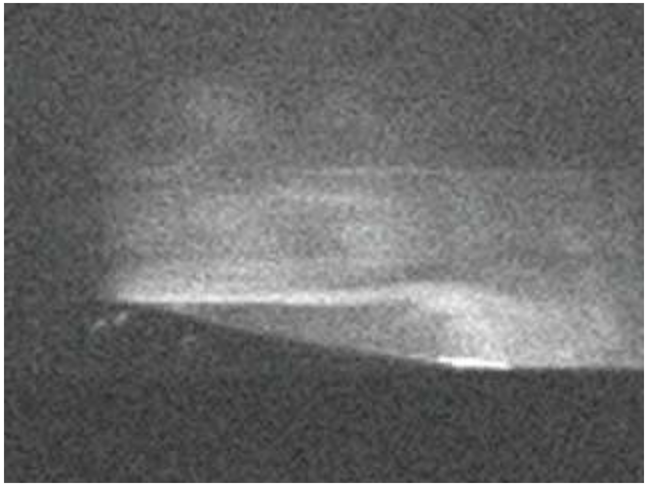
Flow is from right to left



Flow is from right to left



Flow is from left to right

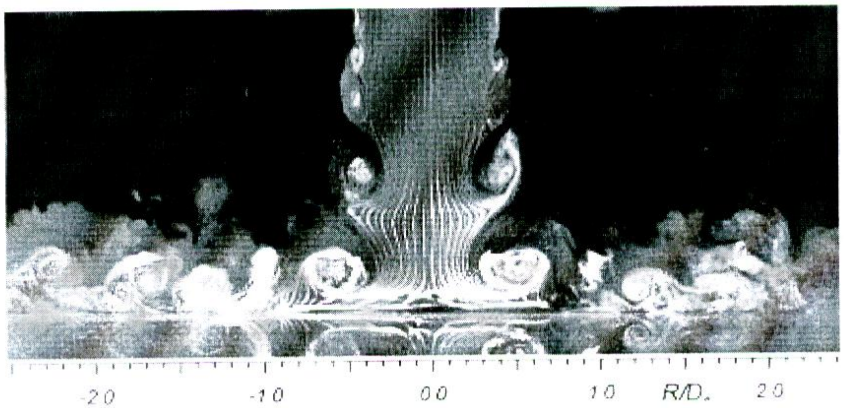


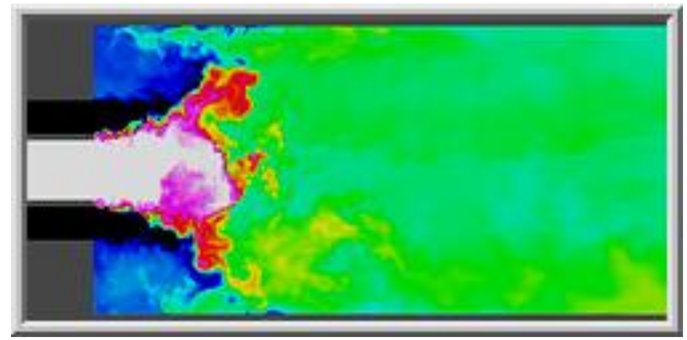
# Turbulence

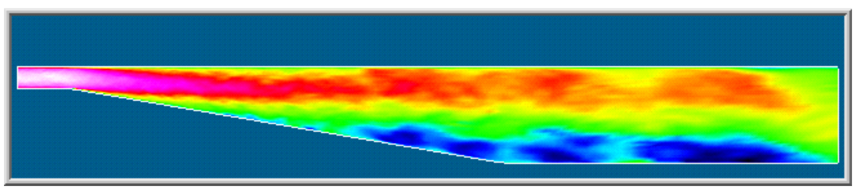
Leading edge flow

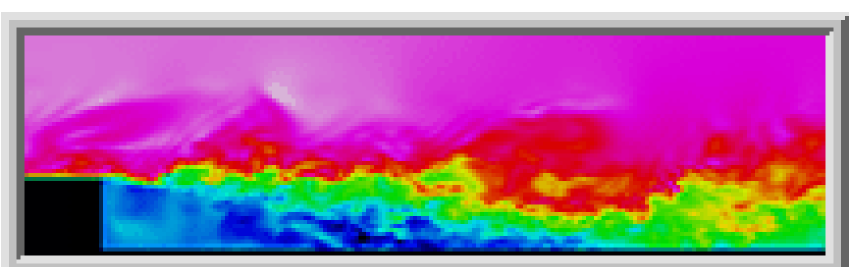
Boundary layer flow

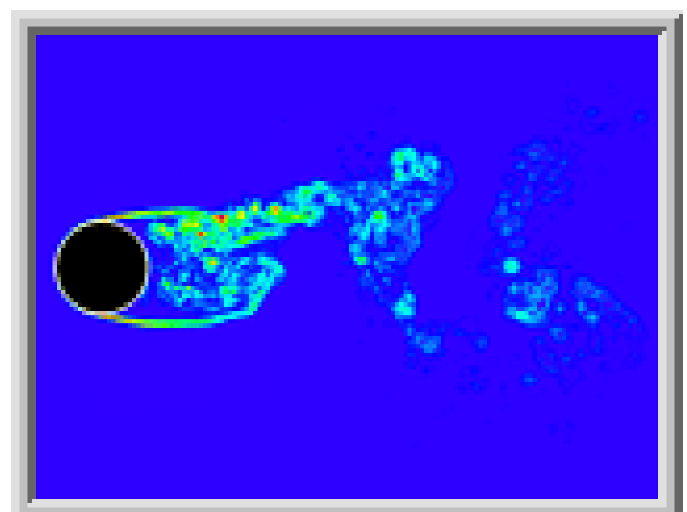
# Turbulence



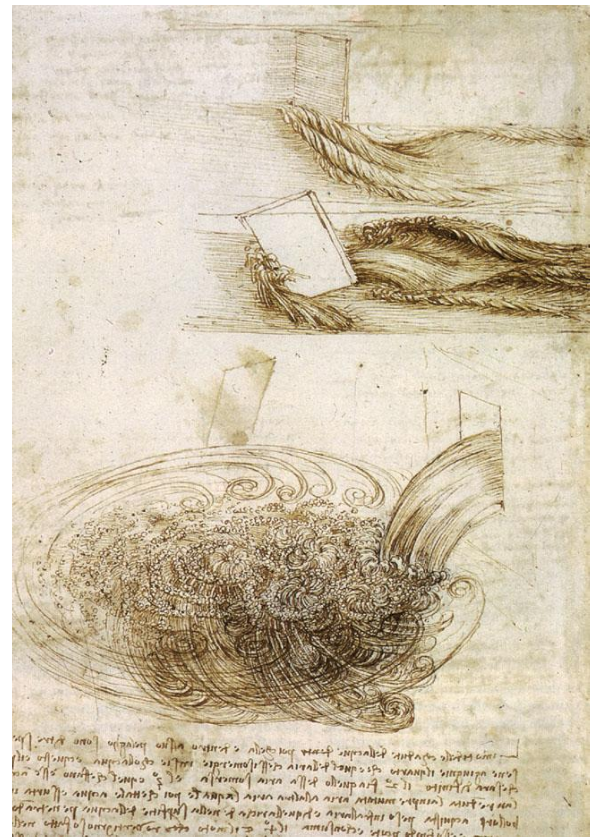







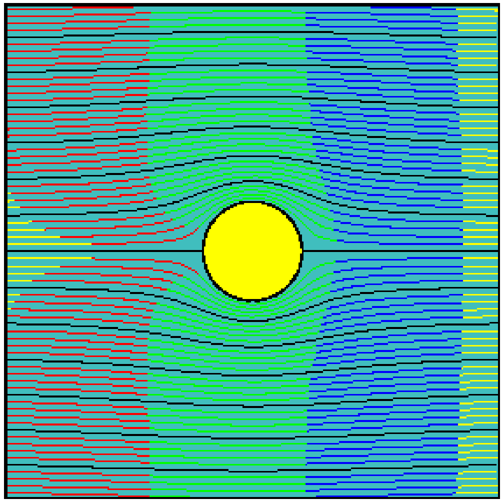


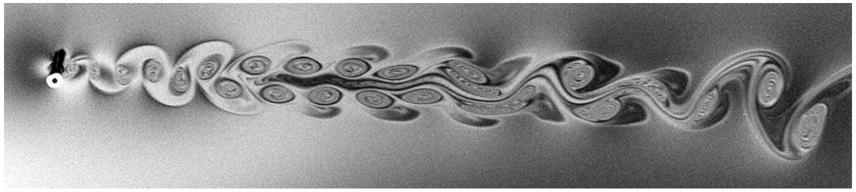
# Chaos

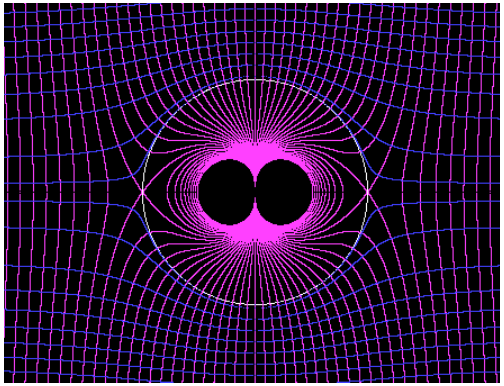


# Paradox d'Alembert

TRAITÉ
DE
DYNAMIQUE,
A PARIS,
M. DCC. LVIII.







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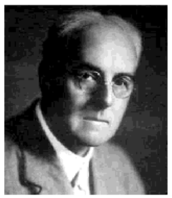
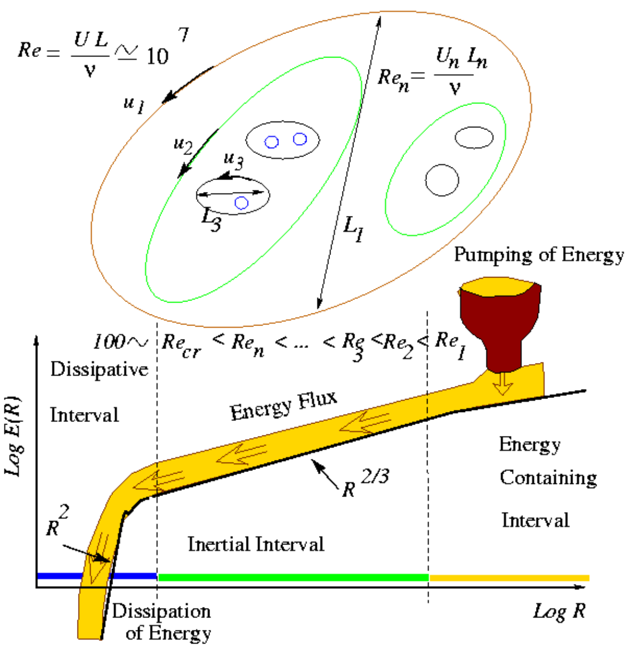
# Richness in scale, high mixing, caos in time and in space, the energy cascade

**Richardson** (1922) **Kolmogorov** (1941)



**Picture of Turbulence**



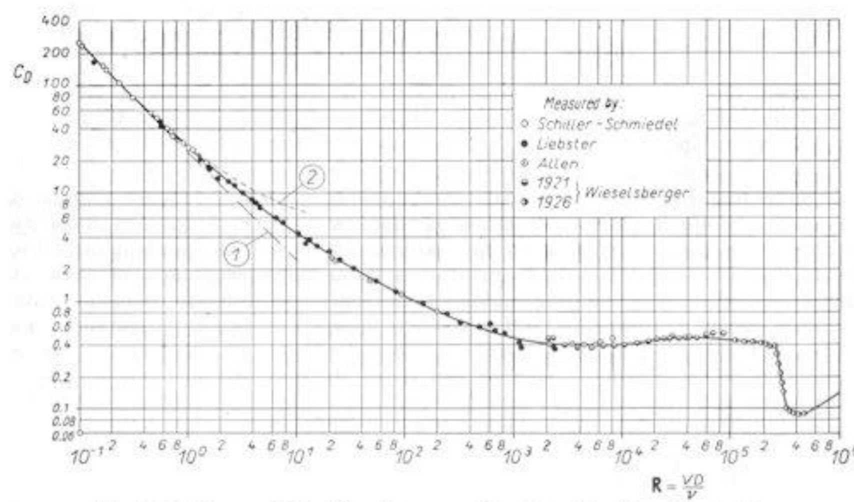
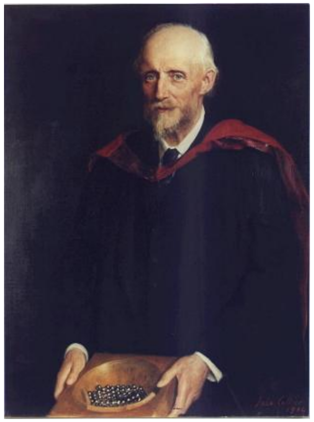




Fig. 1.5. Drag coefficient for spheres as a function of the Reynolds number

Curve (1): Stokes' theory, eqn. (6.10); curve (2): Oseen's theory, eqn. (6.13)

# Historical Perspective to Turbulence

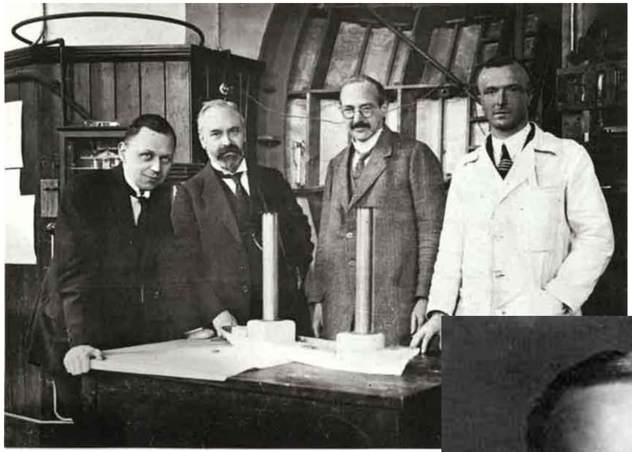


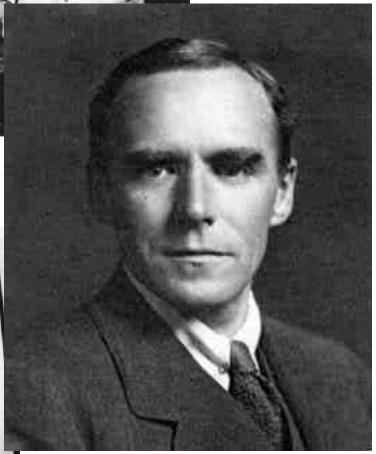
**Osbone Reynolds**
**1883-1898**

**Boussinesq**
**1877-1890**

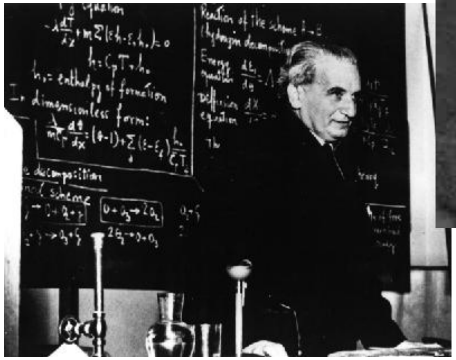
**Saint-Venant**
**1830-1845**

**Prandtl and Ackeret (1904-1930)**





**G.I. Taylor**
**1917-1956**



**T. Von Karman (1933-1949)**

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# Cronology

- 1840 Barré de St. Venant: ($\Delta P \sim AQ + BQ^2$).
- 1854 Hagen: first vizualization studies.
- 1877 Boussinesq: eddy viscosity.
- 1883 Reynolds: Reynolds number.
- 1894 Reynolds: decomposition of the flow properties into mean and fluctuating quantities.
- 1904 Prandtl: boundary layer theory.
- 1916 Taylor: mixing-length, law of the wall.
- 1915-1935 Taylor – von Kármán: statistic theories.

# Cronology (continued)

| | | |
|---|---|---|
| 1939 | Millikan: | logarithmic law of the wall. |
| 1941 | Kolmogorov: | turbulence cascade, -5/3. |
| 1942 | Kolmogorov: | two-equation differential models. |
| 1945 | Chou: | Reynolds stress models (RSM). |
| 1956 | Clauser,Coles: | law of the wake. |
| 1967 | Kovasznay: | turbulent boundary layer asymptotic structure. |
| 1969, 1972 | Yajnik, Mellor: | turbulent boundary layer asymptotic structure. Matched asymptotic expensions method. |
| Presently | Differential models, RSM, LES. | |

# Averaging procedure

Equations of motion

- $U = \overline{U} + u$
- Time average, space average, ensemble average.

$$\frac{\partial \overline{u_i}}{\partial x_i} = 0$$

Continuity equation

$$\frac{\partial \overline{u_j}}{\partial t} + u_i \frac{\partial \overline{u}_j}{\partial x_i} = -\frac{1}{\rho}\frac{\partial \overline{p}}{\partial x_i} + \upsilon \frac{\partial}{\partial x_j}\left(\frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial \overline{u_j}}{\partial x_i}\right) - \frac{\partial}{\partial x_j}\left(\overline{u'_i u'_j}\right)$$

Momentum equation

inertia effects

pressure effects

viscous effects

turbulent effects

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# Modelling strategies, eddy viscosity, κ–ε model, RSM

## RANS Equations

$$\frac{\partial \overline{u_i}}{\partial x_i} = 0$$

$$\frac{\partial \overline{u_j}}{\partial t} + u_i \frac{\partial \overline{u_j}}{\partial x_i} = -\frac{1}{\rho}\frac{\partial \overline{p}}{\partial x_i} + \upsilon \frac{\partial}{\partial x_j}\left(\frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial \overline{u_j}}{\partial x_i}\right) - \frac{\partial}{\partial x_j}\left(\overline{u'_i u'_j}\right)$$

Term to be modelled

fluctuating quantity, turbulent stresses

## I) Eddy viscosity hypothesis

$$-\overline{u_i' u_j'} = \nu_t \left(\frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial \overline{u_j}}{\partial x_i}\right) - \frac{2}{3}\kappa\delta_{ij}$$

## II) κ–ε model

$$\frac{\mu_t}{\rho} = \nu_t = C_\mu \frac{\kappa^2}{\varepsilon}$$

$$\frac{\partial (U_i \kappa)}{\partial x_i} = \frac{\partial}{\partial x_i}\left(\frac{\nu_t}{\sigma_k}\frac{\partial \kappa}{\partial x_i}\right) - \overline{u_i u_j} S_{ij} - \varepsilon$$

**Turbulent Kinetic Energy**

$$\frac{\partial (U_i \varepsilon)}{\partial x_i} = \frac{\partial}{\partial x_i}\left(\frac{\nu_t}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial x_i}\right) - \frac{\varepsilon}{\kappa}\left(C_1 \overline{u_i u_j} S_{ij} + C_2 \varepsilon\right)$$

**Dissipation Rate by Mass Unit**

# Reynolds stress models

## III) RSM

**Transport equation for Reynolds stress tensor elements**

$$\frac{\partial \tau_{ij}}{\partial t} + \frac{\partial \bar{u}_k \tau_{ij}}{\partial x_k} = P_{ij} + \frac{2}{3}\beta' \omega \kappa \delta_{ij} + \frac{\partial}{\partial x_k}\left(\left(\upsilon + \frac{\upsilon_t}{\sigma^*}\right)\frac{\partial \tau_{ij}}{\partial x_k}\right)$$

**Production**

$$P_k = -\overline{u_i' u_j'}\frac{\partial \overline{u}_j}{\partial x_i}$$

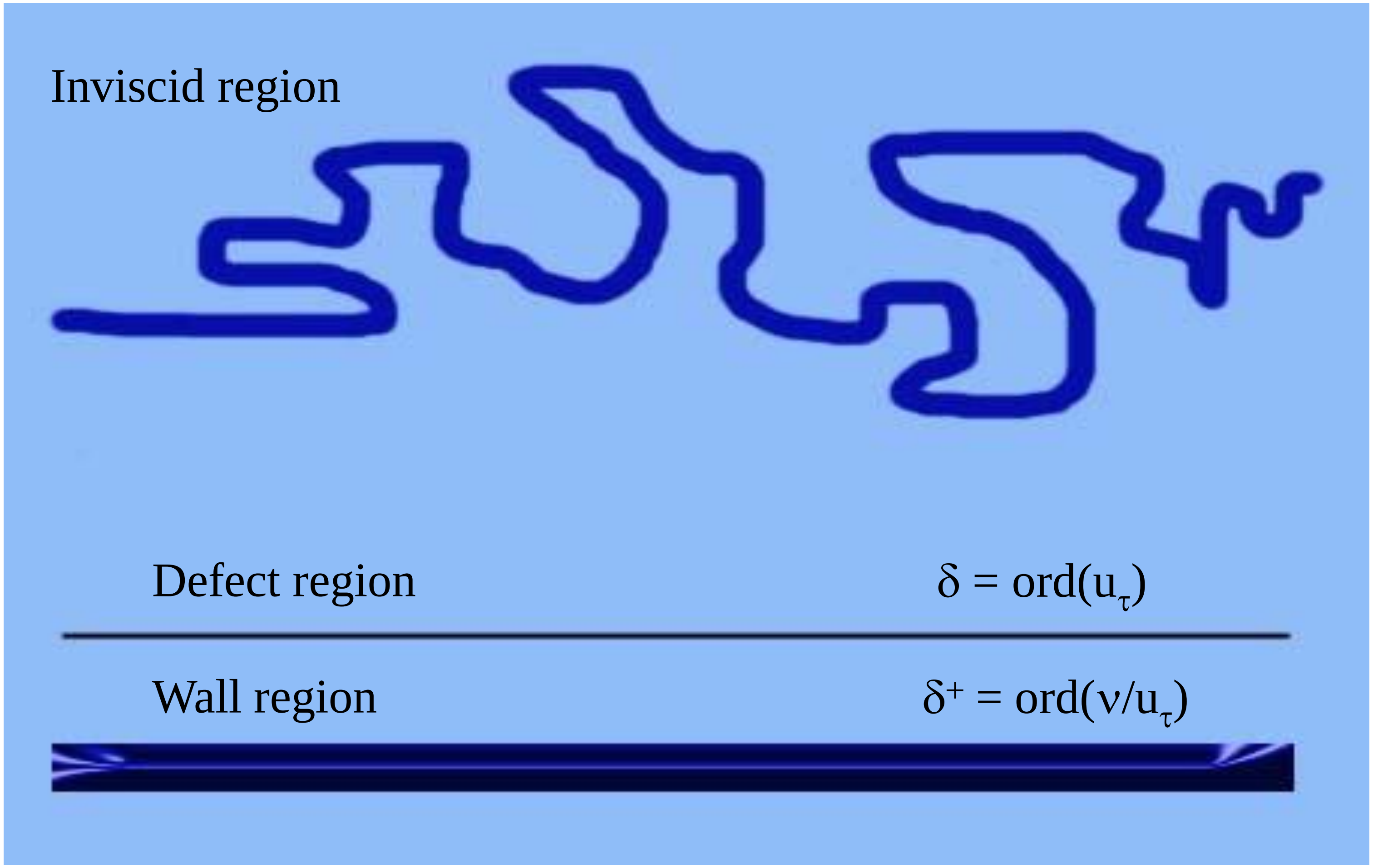
**Transportable quantity**

$$\frac{\partial \omega}{\partial t} + \frac{\partial \bar{u}_k \omega}{\partial x_k} = \alpha_3 \frac{\omega}{\kappa} P_\kappa - \beta_3 \omega^2 + \frac{\partial}{\partial x_k}\left(\left(\upsilon + \frac{\upsilon_t}{\sigma_{\omega 3}}\right)\frac{\partial \omega}{\partial x_k}\right) + (1 - F_1)^2 \frac{1}{\sigma_2 \omega}\frac{\partial \kappa}{\partial x_k}\frac{\partial \omega}{\partial x_k}$$

Model relies on many empirical constants

$\sigma^* = 2$ $\qquad$ $\alpha = \frac{\beta}{\beta'} - \frac{k^2}{\sigma(\beta')^{0.5}} = 5/9$ $\qquad$ $\beta = 0.075$ $\qquad$ $k = 0.41$

# Two-layered asymptotic structure



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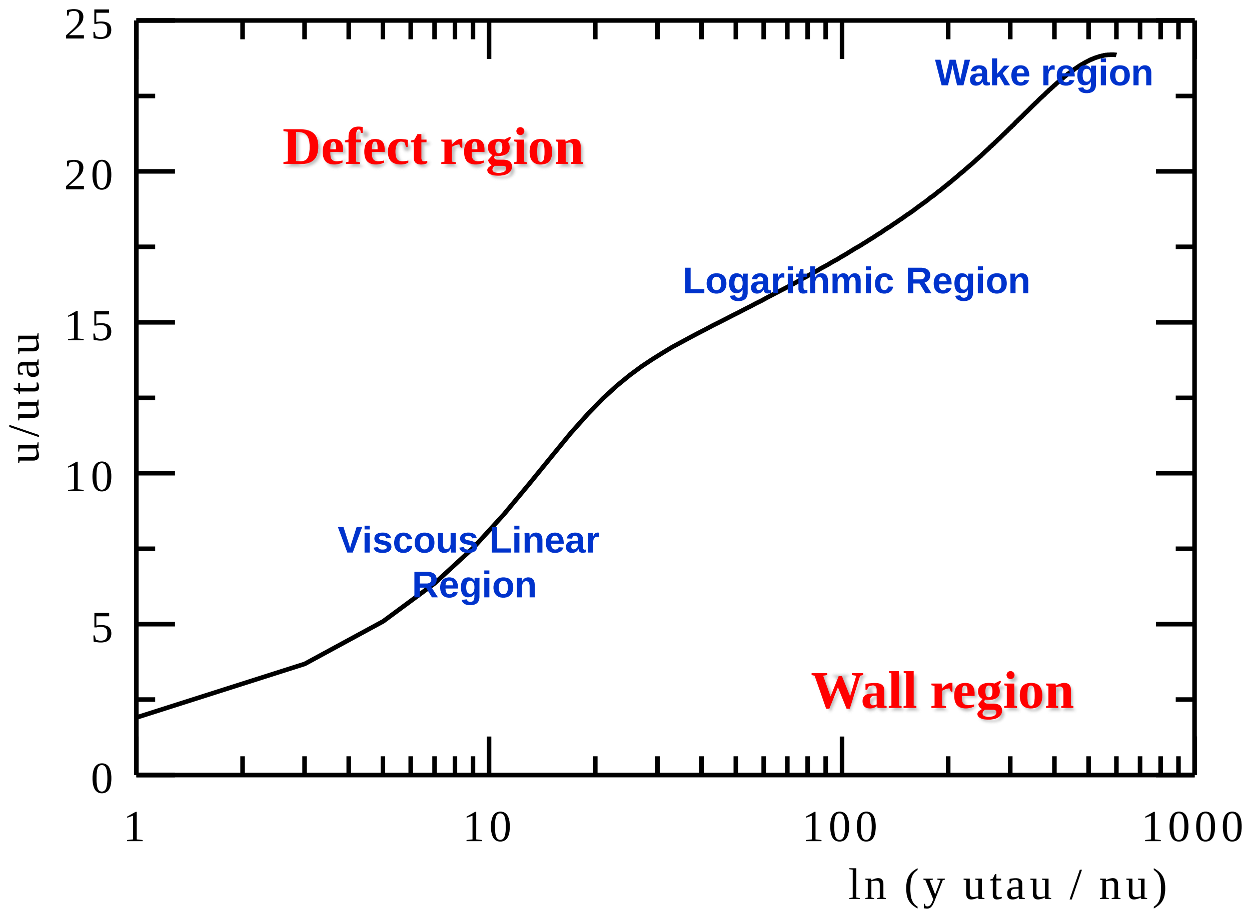
# Two-layered asymptotic structure



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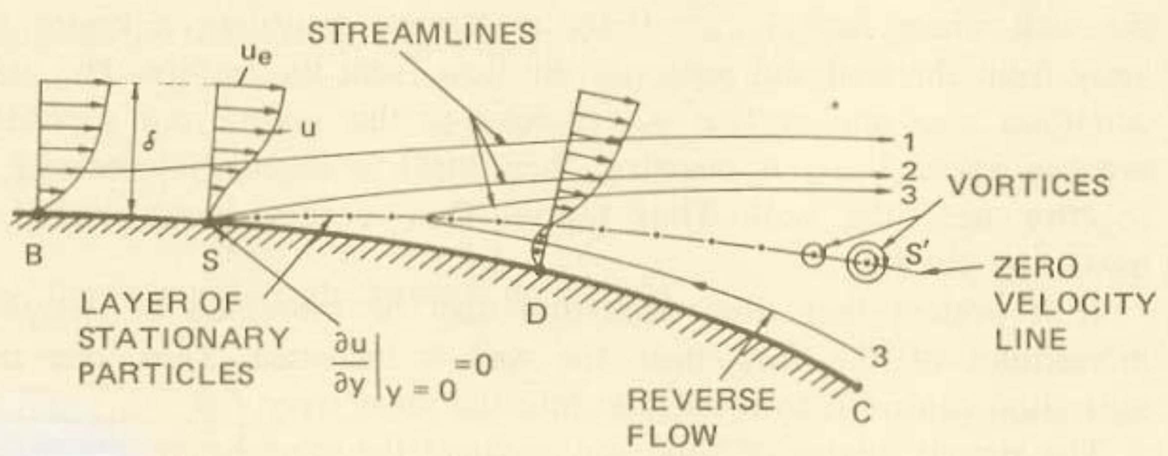
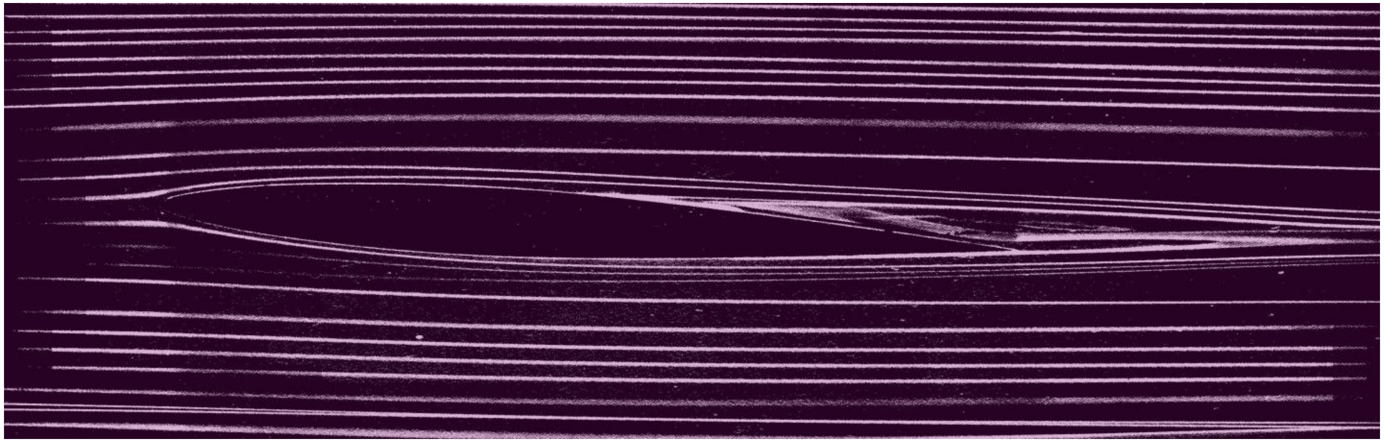
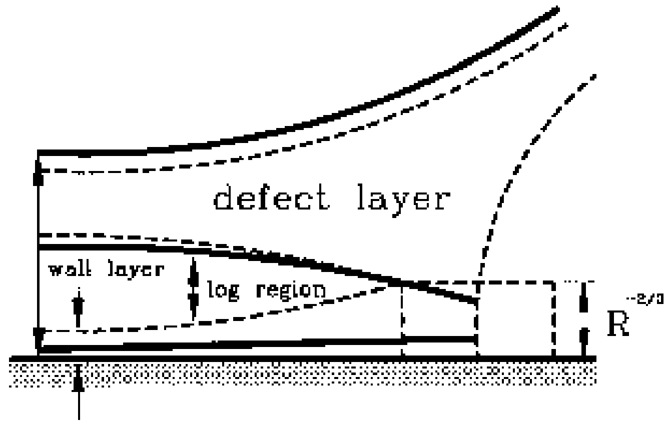
# Changes due to flow separation



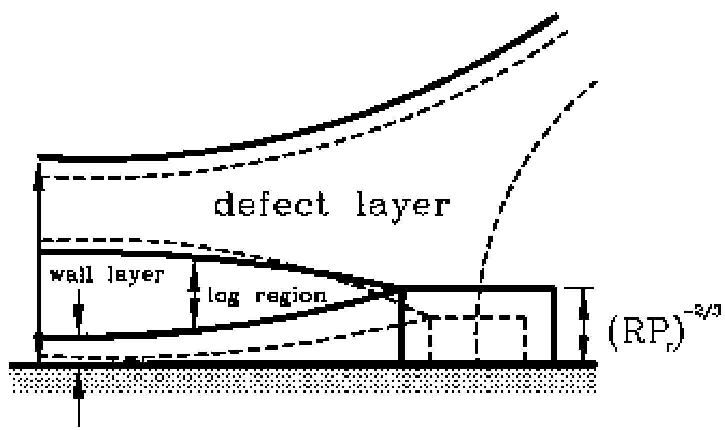

Velocity profile near a separation point.



# Structure near a separation point




a) $P_r > 1$




b) $P_r < 1$

# Law of the wall Formulation

- **Mellor (1966)**

$$u^+ = z^+ + \frac{1}{2} p^+ z^{+2}$$

$$u^+ = \xi_{p^+} + \frac{2}{\kappa}\left(\sqrt{1+p^+z^+} - 1\right) + \frac{1}{\kappa}\left(\frac{4z^+}{2+p^+z^+ + 2\sqrt{1+p^+z^+}}\right)$$

$$u^+ = u / u_{p\nu}$$
$$z^+ = z u_{p\nu} / \nu$$
$$p^+ = [(\nu/\rho)(dp/dx)] / u_\tau^3$$
$$u_{p\nu} = [(\nu/\rho)(dp/dx)]^{1/3}$$

- **Nakayana & Koyama (1984)**

$$u^+ = \frac{2}{\kappa^+}\left[3(\varsigma - \varsigma_S) + \ln\left(\frac{\varsigma_S + 1}{\varsigma_S - 1}\frac{\varsigma - 1}{\varsigma + 1}\right)\right]$$

$$u^+ = u / u_\tau \qquad \varsigma = \left(\frac{1+2\tau^+}{3}\right)$$

$$\tau^+ = 1 + p^+ z^+$$
$$z^+ = (\tau_w / \rho)^{1/2} z / \nu$$
$$\kappa^+(p^+) = \frac{0.419 + 0.539 p^+}{1 + p^+}$$
$$\varsigma_S(p^+) = (1 + 0.074 p^+)^{1/2}$$

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# Law of the wall Formulation

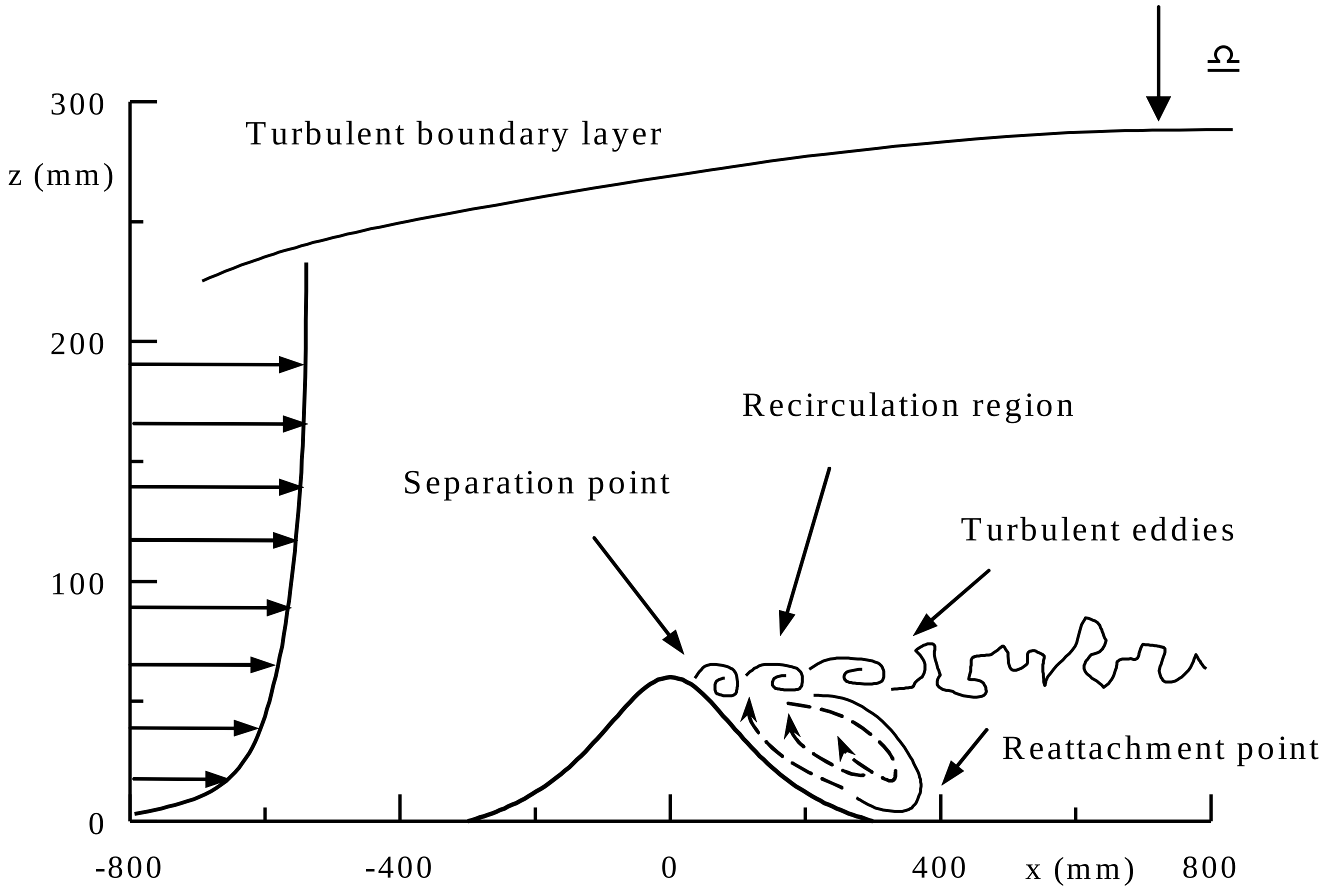
- **Cruz and Silva Freire (1998, 2002)**

$$u = \frac{\tau_w}{|\tau_w|}\frac{2}{\kappa}\sqrt{\frac{\tau_w}{\rho} + \frac{1}{\rho}\frac{dP_w}{dx} z} + \frac{\tau_w}{|\tau_w|}\frac{u_\tau}{\kappa}\ln\left(\frac{z}{L_c}\right)$$

$$L_C = \frac{\sqrt{\left(\frac{\tau_w}{\rho}\right)^2 + 2\frac{\nu}{\rho}\frac{dP_w}{dx}u_R} - \frac{\tau_w}{\rho}}{\frac{1}{\rho}\frac{dP_w}{dx}}$$

$$u = -\frac{2}{\kappa}u_\tau - \frac{u_\tau}{\kappa}\ln\left(\frac{z}{L_C}\right) \qquad L_C = 2\left|\frac{\tau_w}{dP_w/dx}\right|$$

# Experiments



# Wind Tunnel

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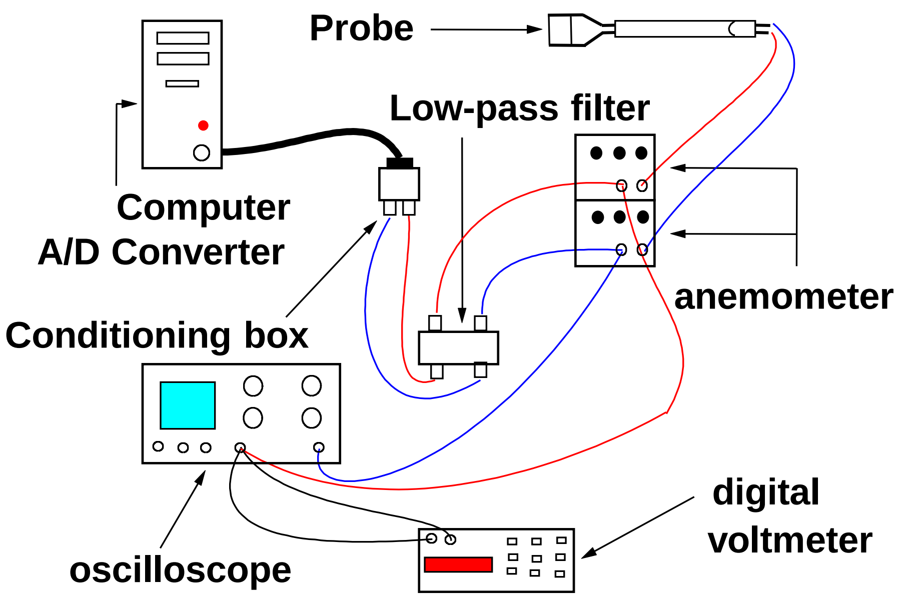
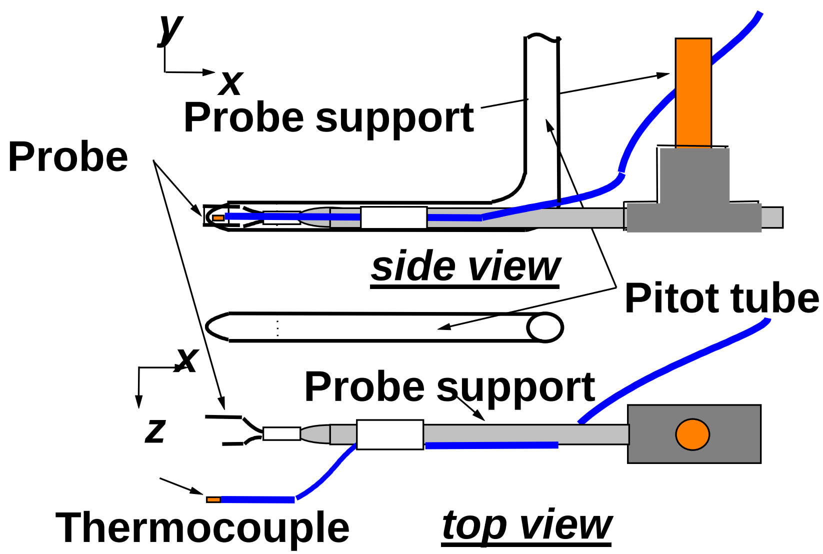
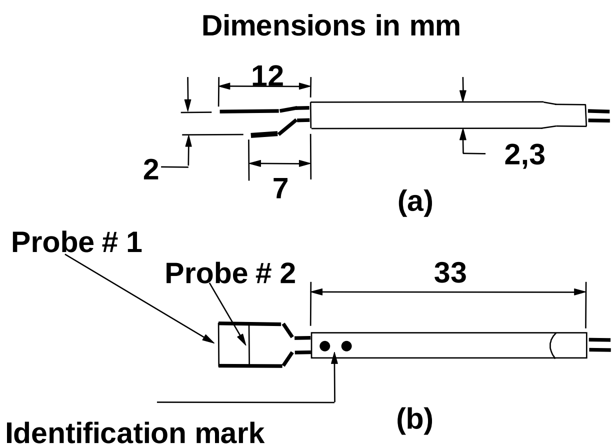
# Hot-wire anemometry

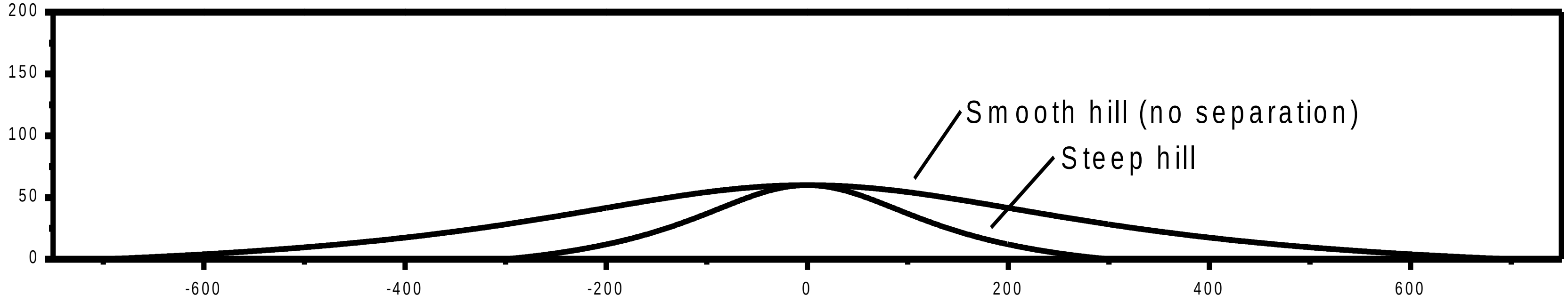



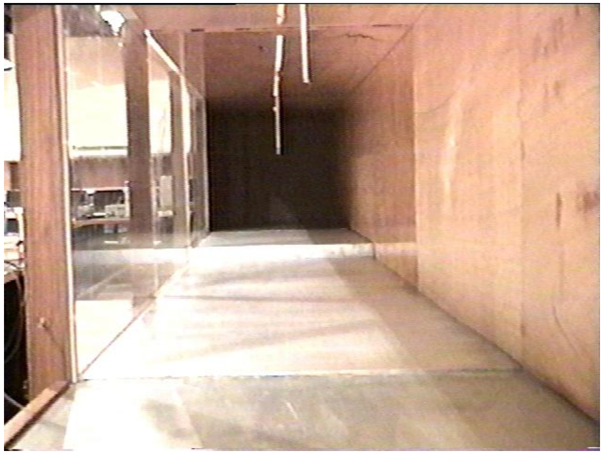
# Hot-wire anemometry

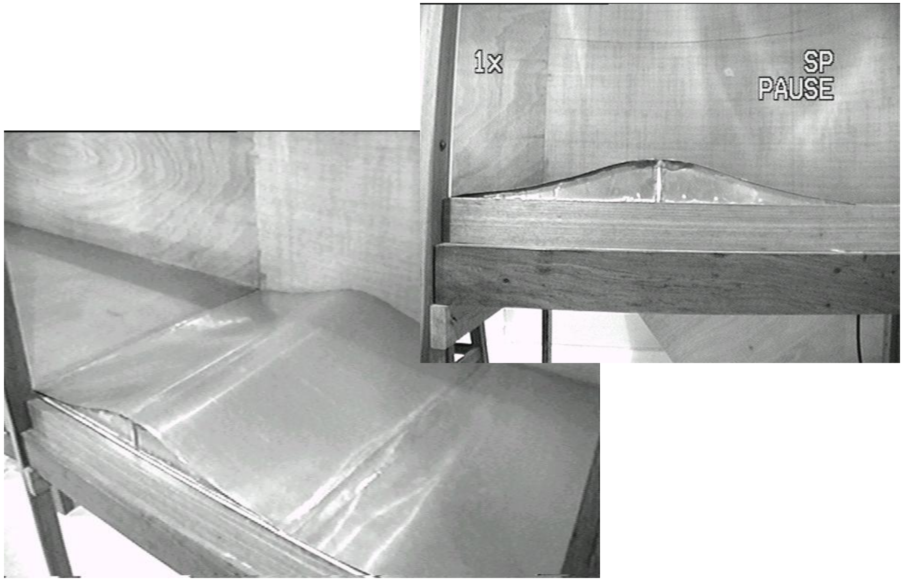


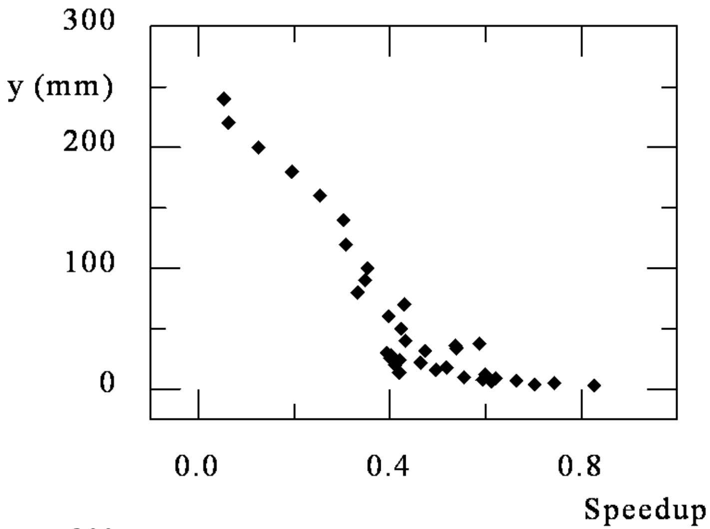
# Hill Model

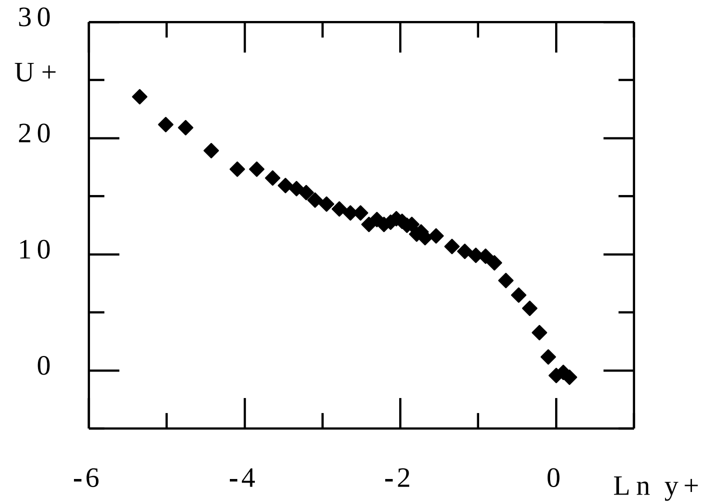

200
150
100
50
0
-600
-400
-200
0
200
400
600
Smooth hill (no separation)
Steep hill

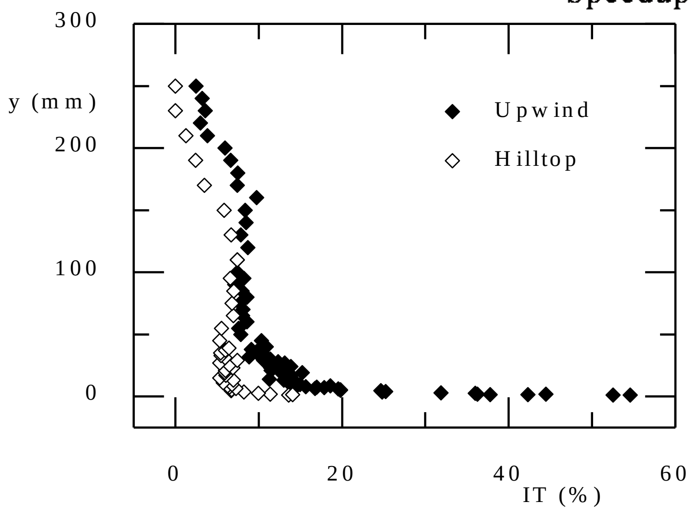


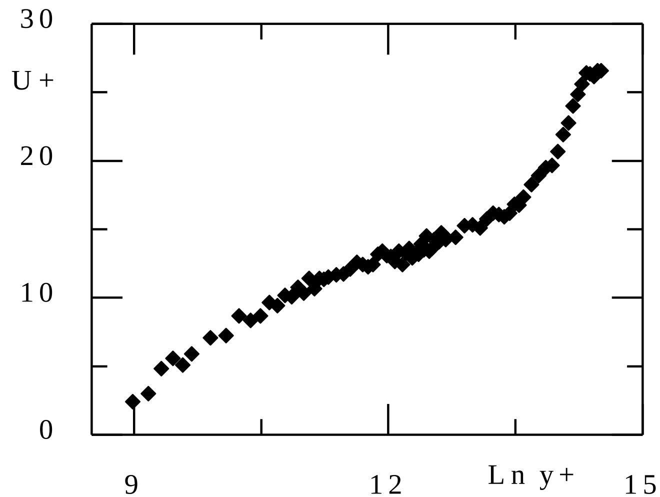

1x
SP
PAUSE

# Results: Velocity profiles



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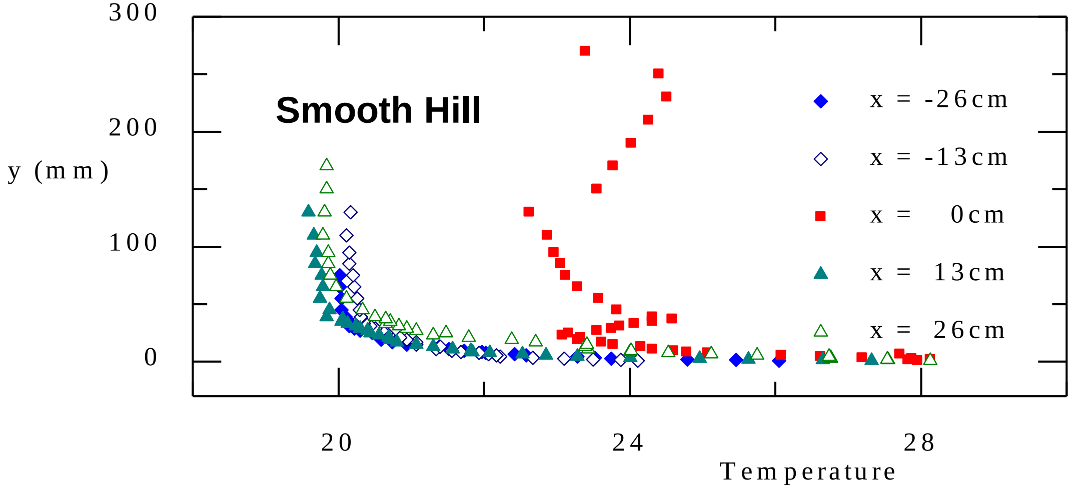
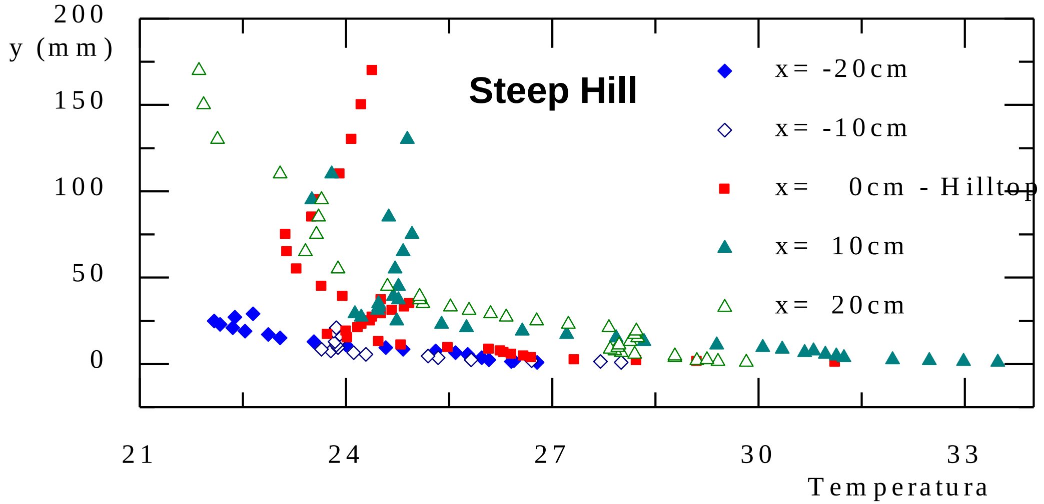
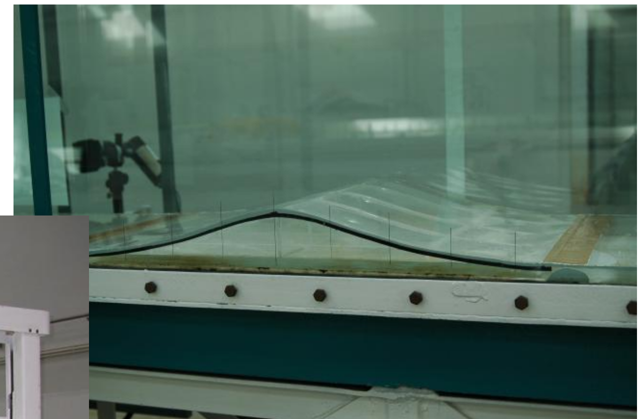
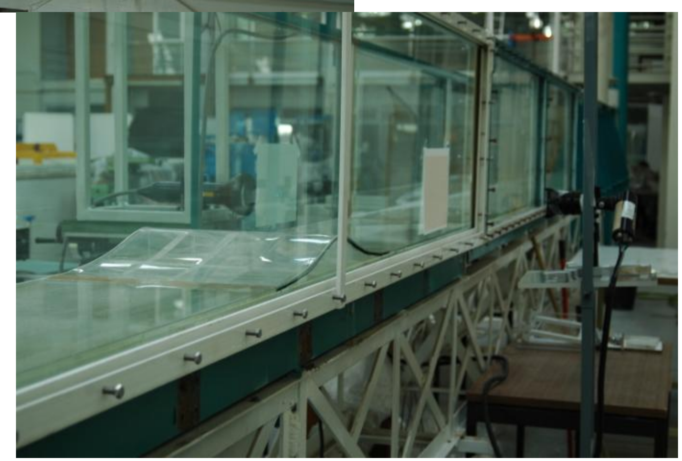
# Results: Temperature profiles



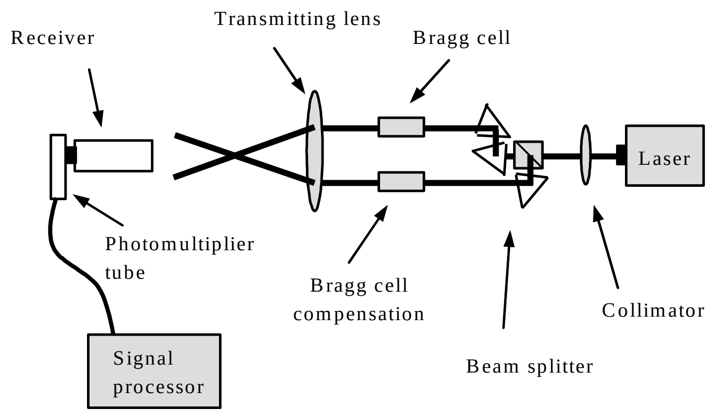



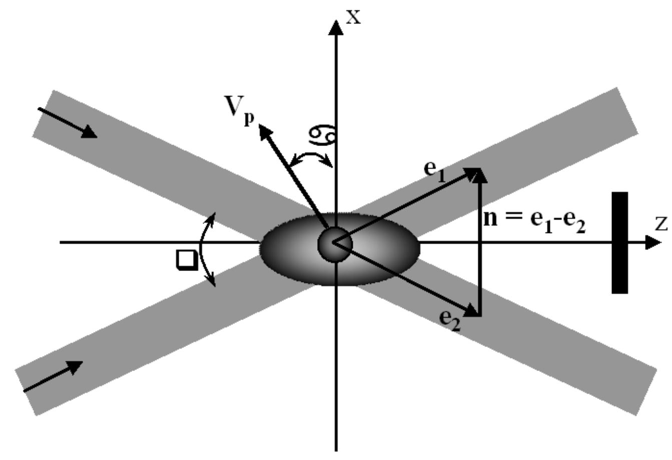
# Water tank and hill model

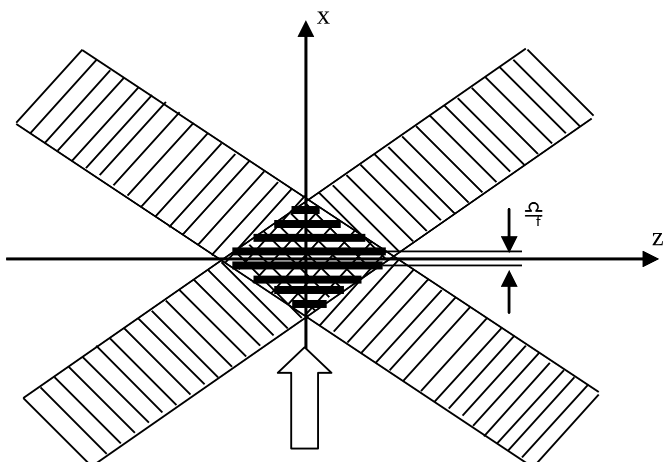


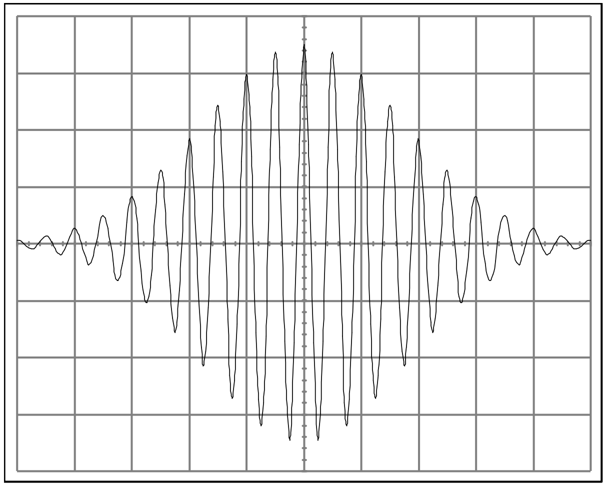


# Laser Doppler Anemometry



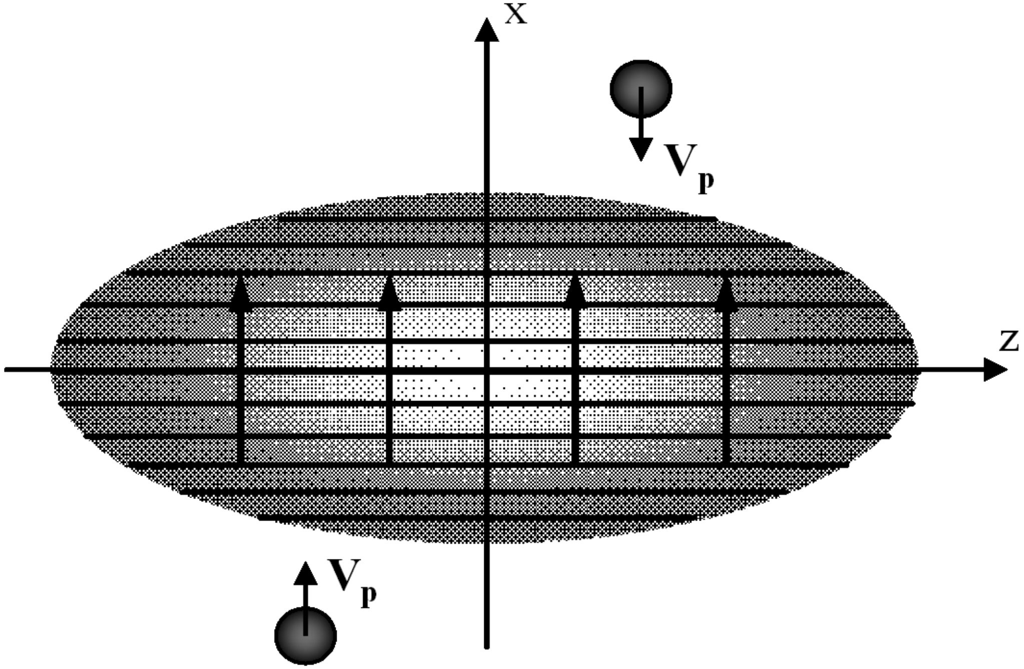



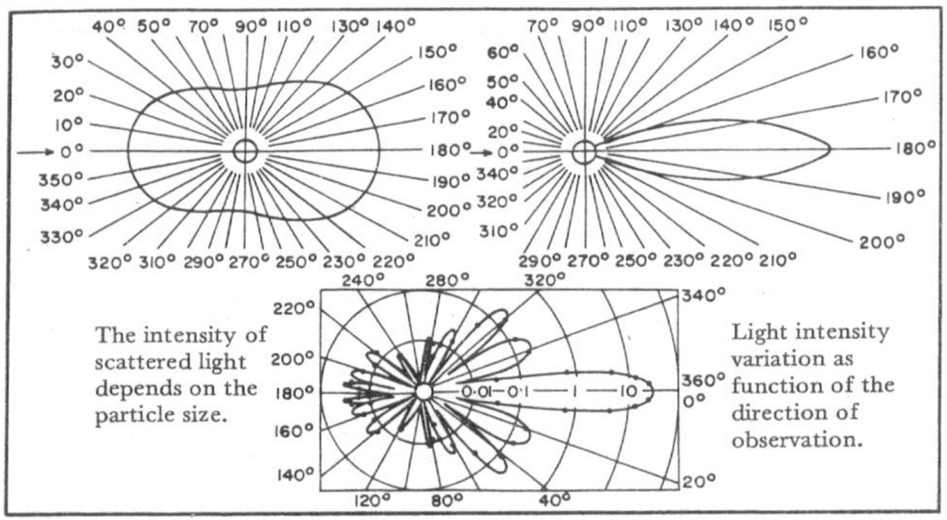
# Laser Doppler Anemometry

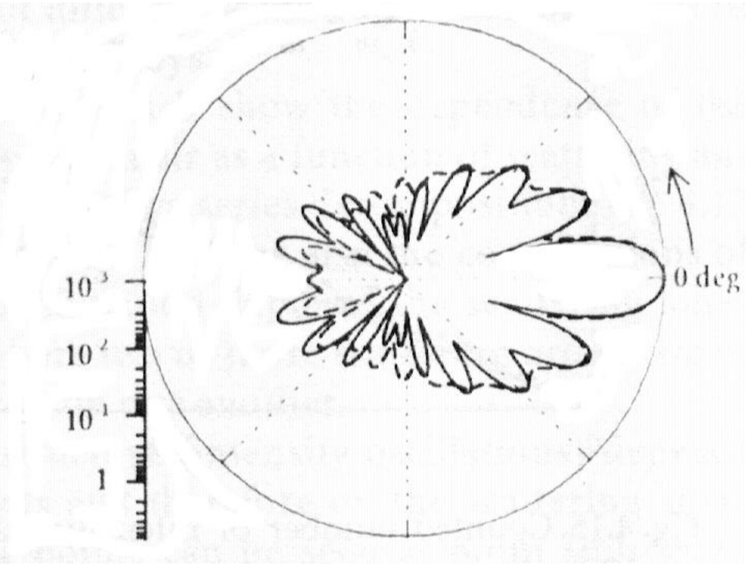
$$f_D = \frac{2\,Sen(\theta/2)}{\lambda_r}\left|V_p\right| Cos\alpha$$

$$f_d = f_{shift} \pm f_D$$

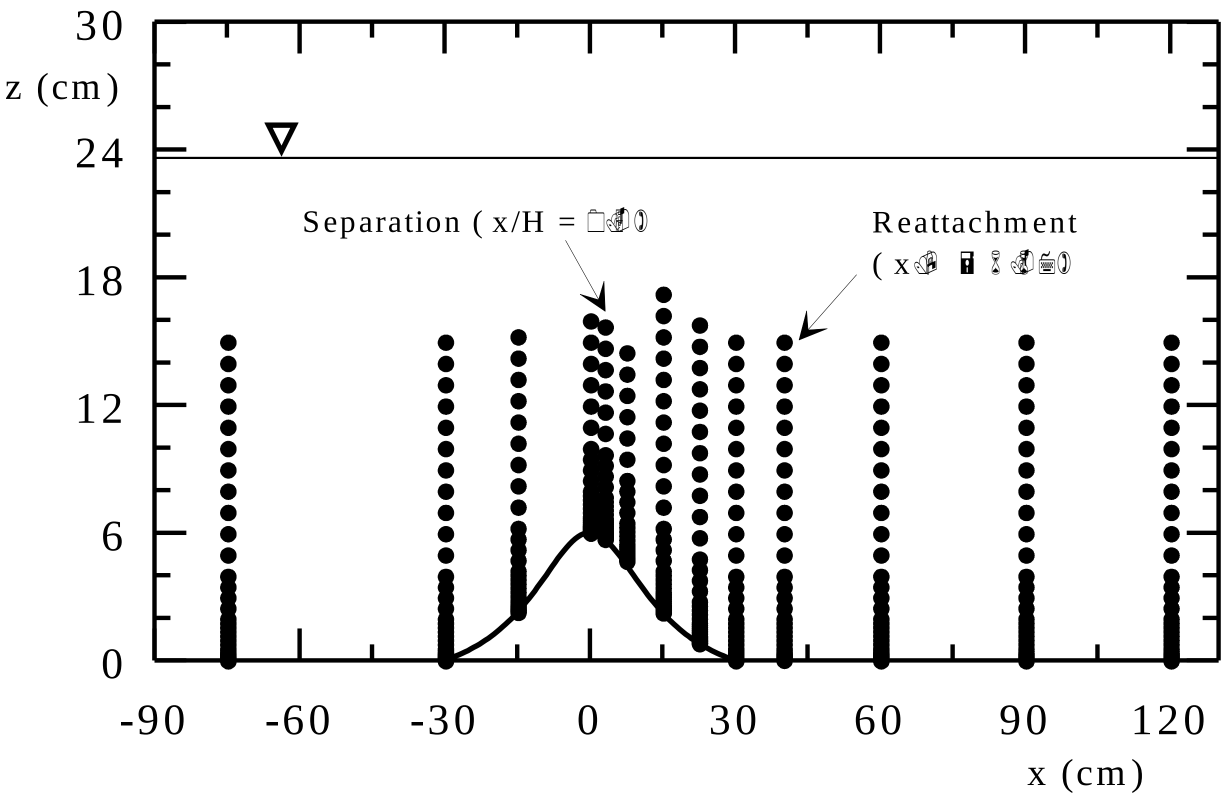


# Laser Doppler Anemometry



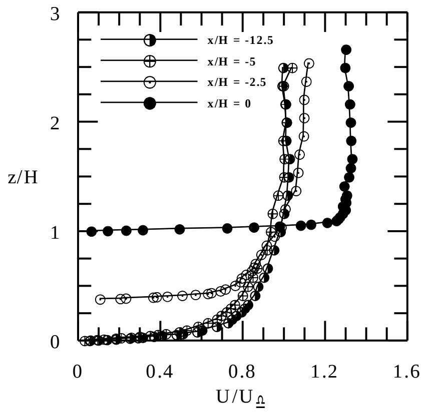



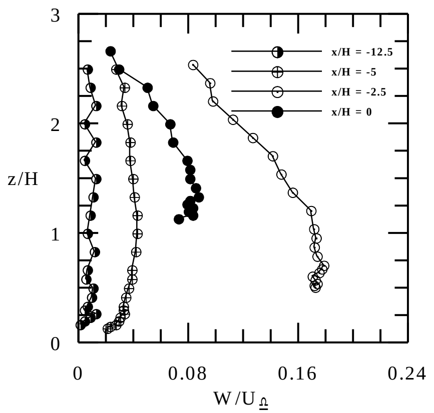
# Measuring stations



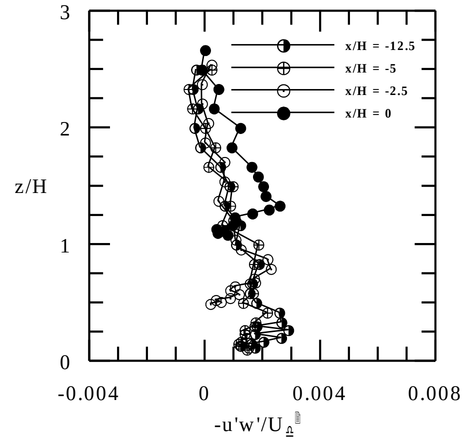
# Results. Upstream region

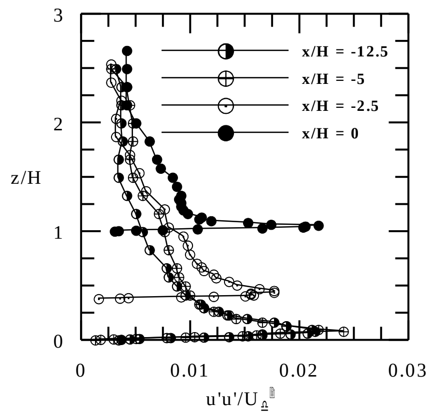
## Mean velocity

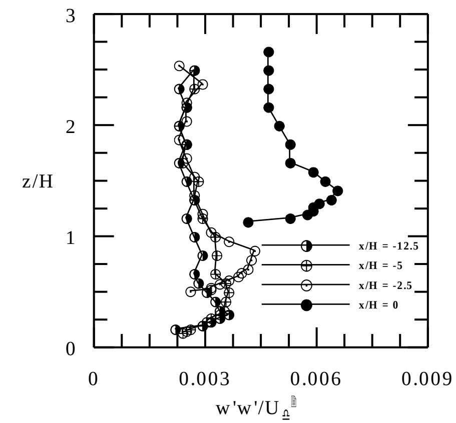



## Turbulent stress

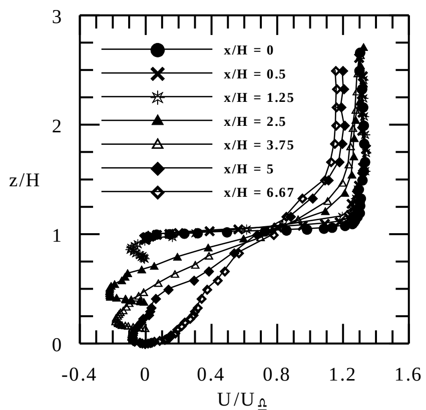


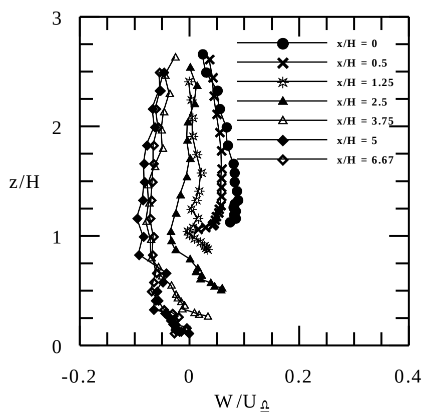



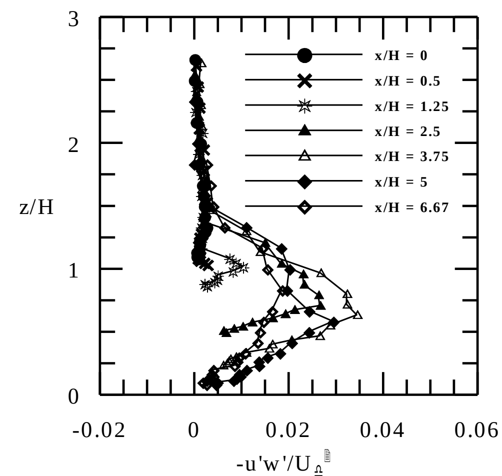
# Results. Separation region

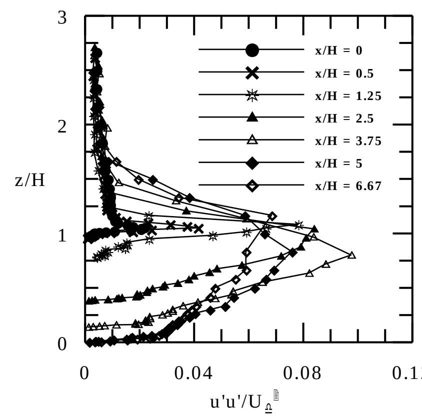
## Mean velocity

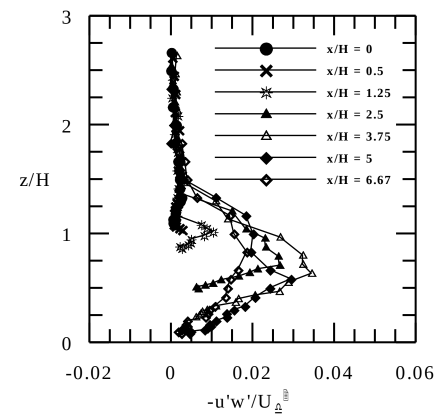



## Turbulent stress

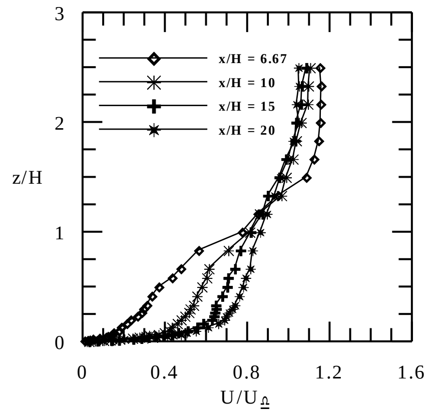


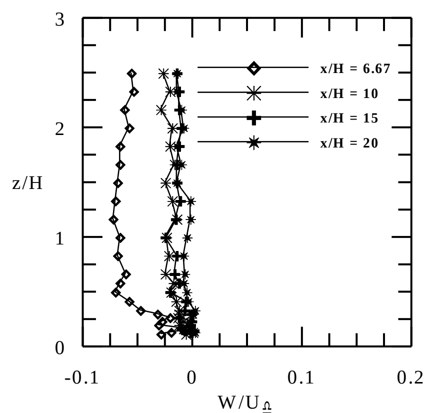



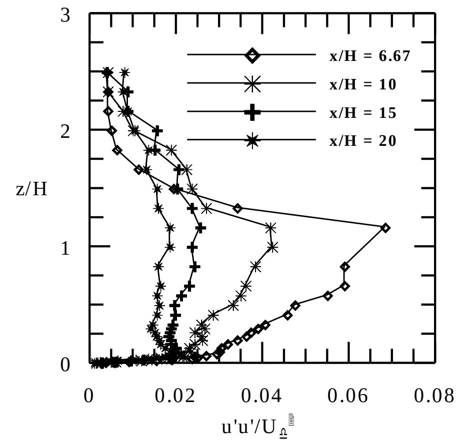
# Results. Downstream region

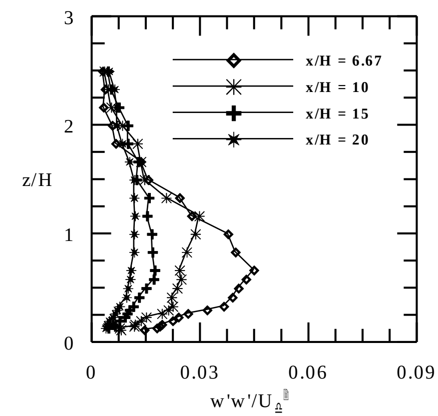
## Mean velocity

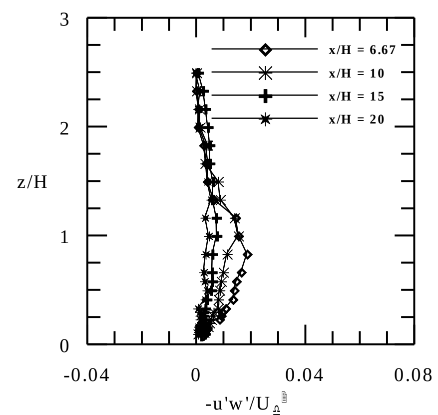



## Turbulent stress



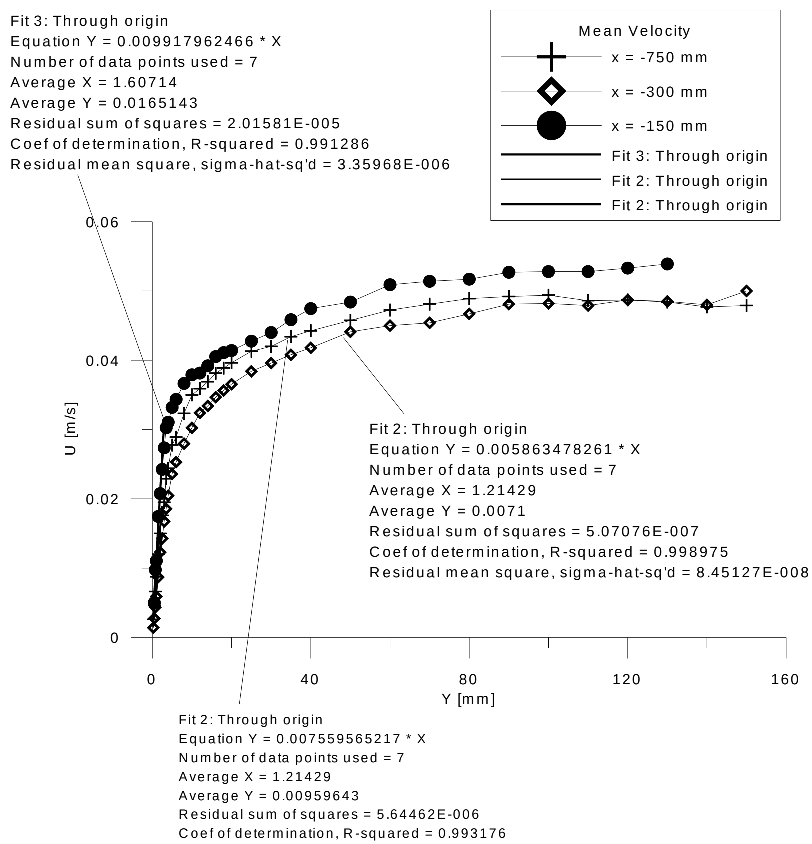



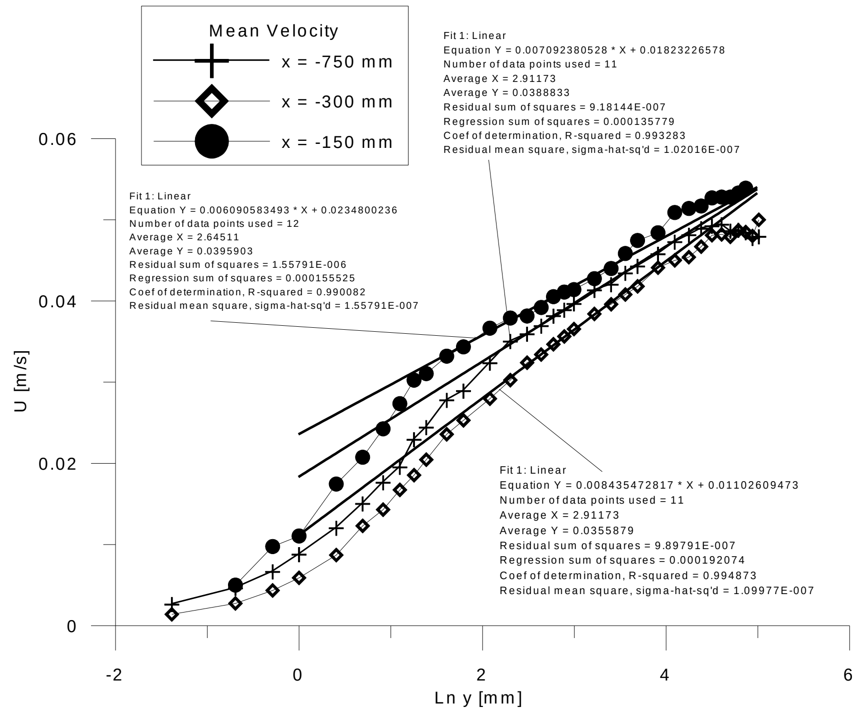
LABORATÓRIO DE MECÂNICA DA TURBULÊNCIA

# Skin-friction results: Upstream region

## Linear Law

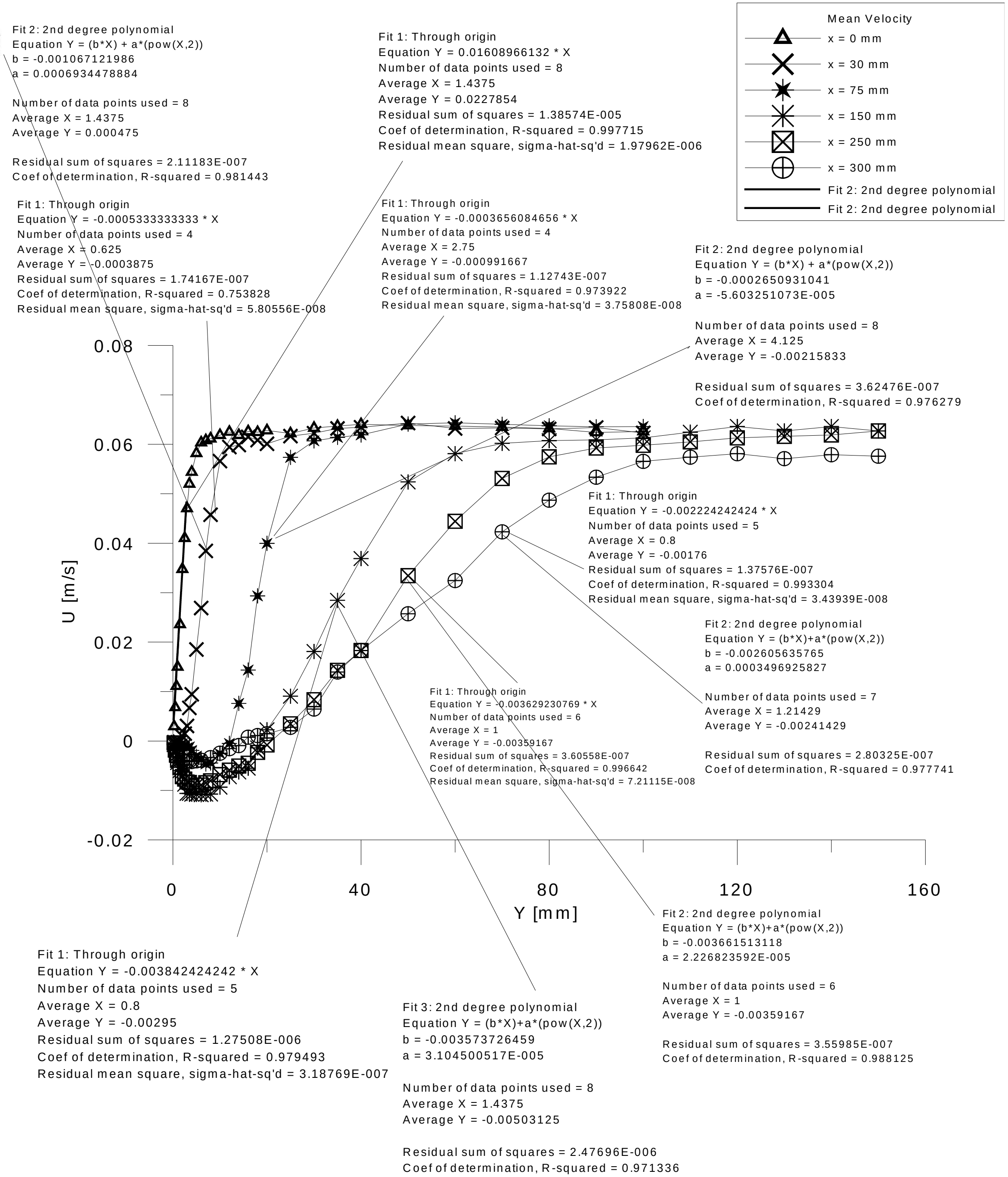



## Log law



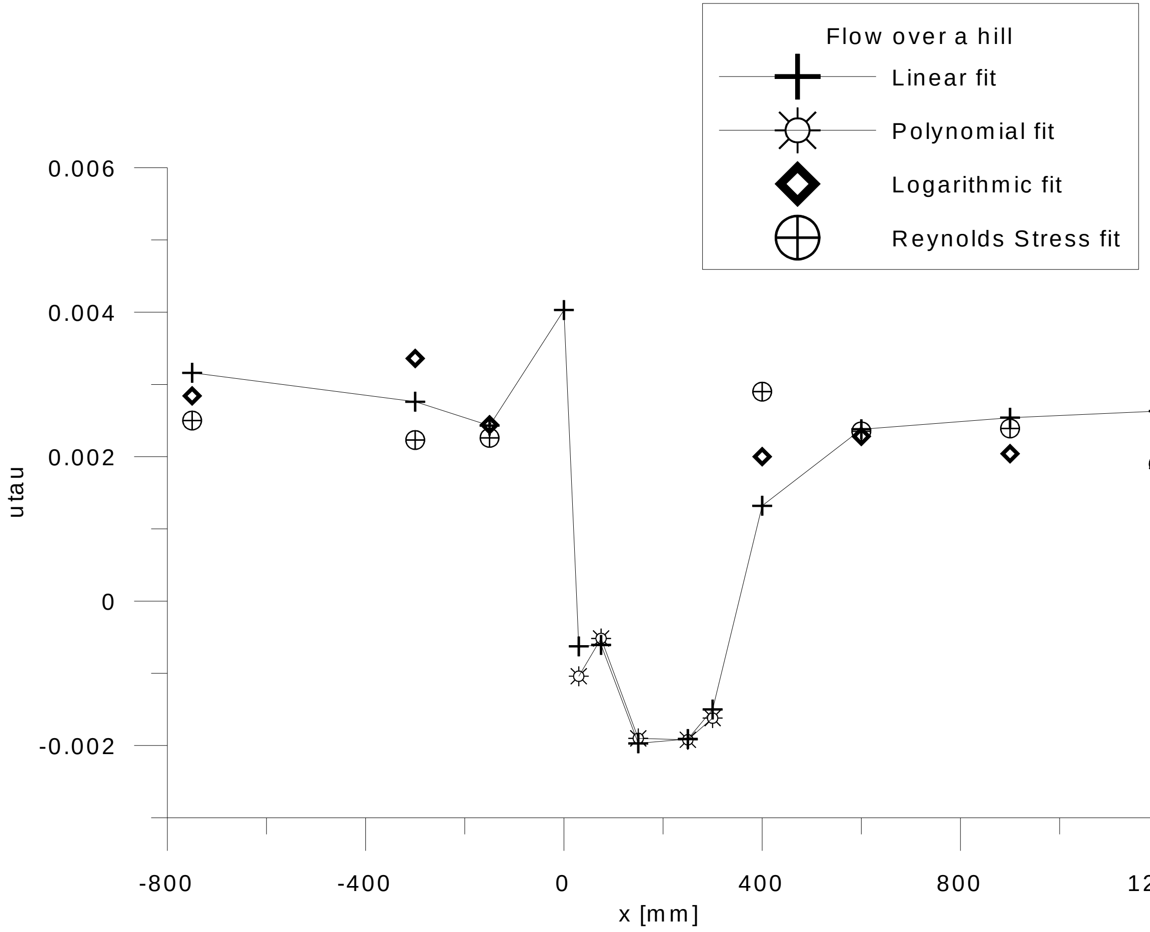
LABORATÓRIO DE MECÂNICA DA TURBULÊNCIA

# Skin-friction results: Upstream region



LABORATÓRIO DE MECÂNICA DA TURBULÊNCIA

# Skin-friction: Consolidated results



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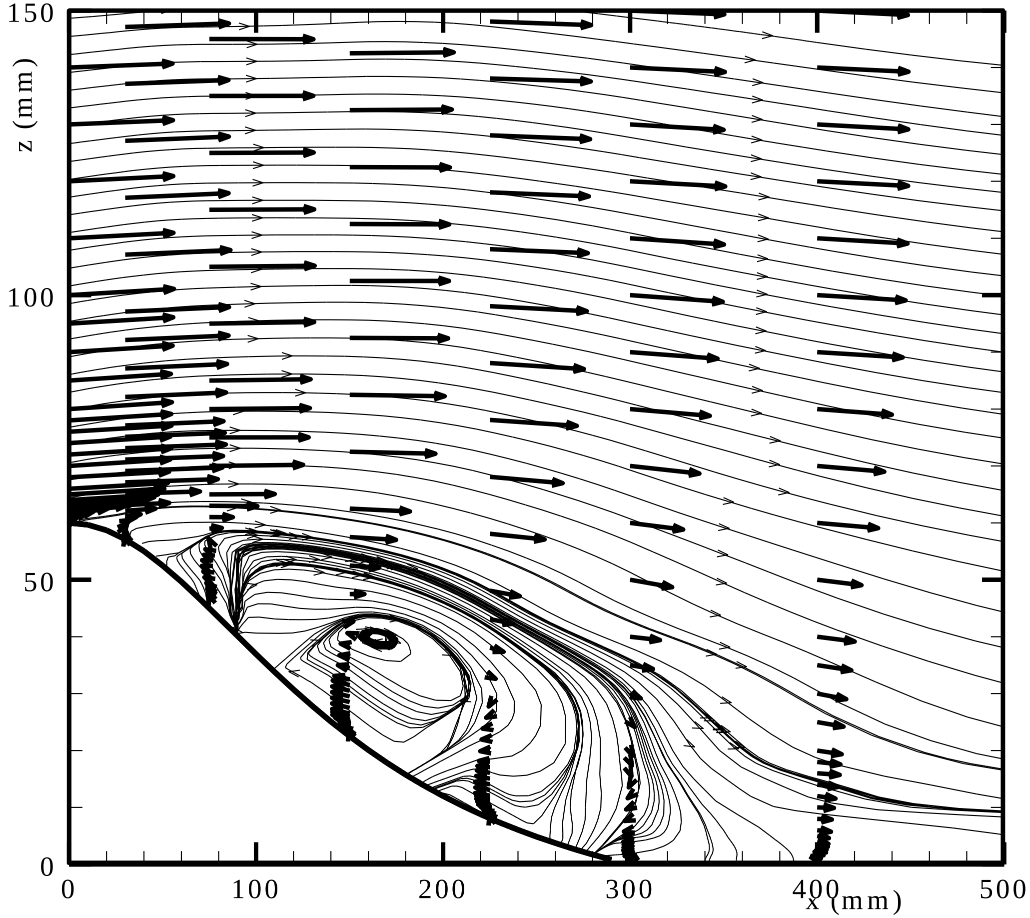
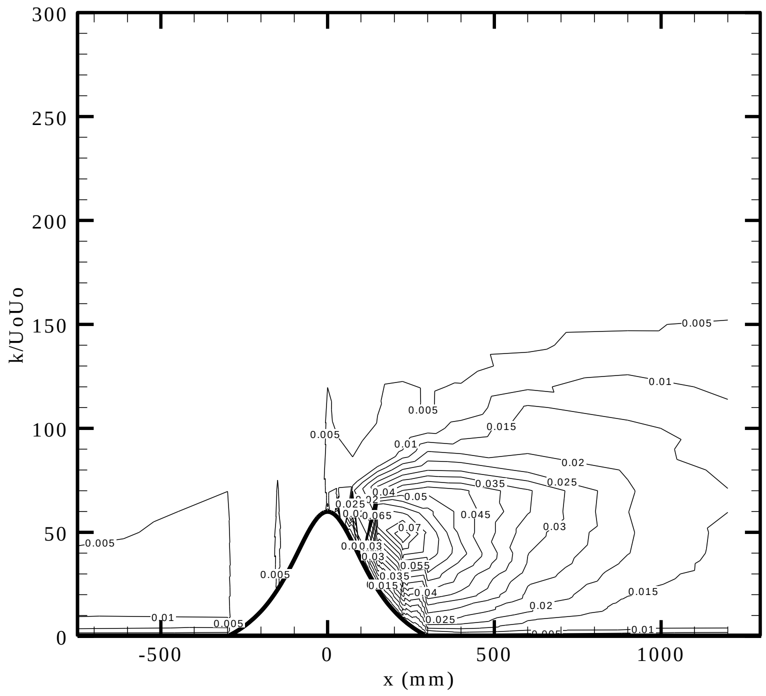
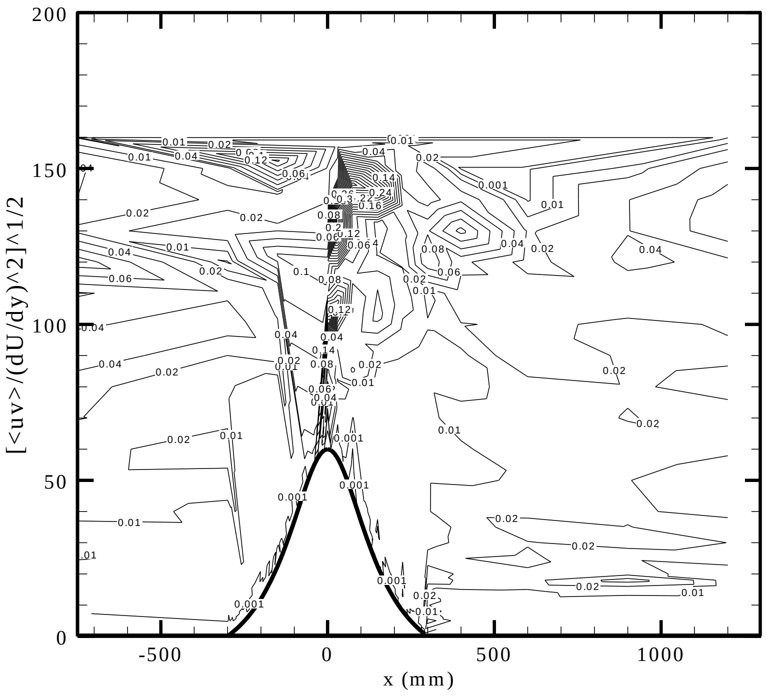
# Streamlines



# Kinetic energy and mixing length



# Numerical simulation
# Eddy Viscosity, κ–ε Model

## RANS Equations

$$\frac{\partial \overline{u_i}}{\partial x_i} = 0$$

$$\frac{\partial \overline{u_i}}{\partial t} + u_i \frac{\partial \overline{u_j}}{\partial x_i} = -\frac{1}{\rho}\frac{\partial \overline{p}}{\partial x_i} + \frac{\partial}{\partial x_j}\left(\upsilon \frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial \overline{u_j}}{\partial x_i}\right) - \frac{\partial}{\partial x_j}\left(\overline{u'_i u'_j}\right)$$

term to be modelled

fluctuating quantity, turbulent stresses

## I) Eddy viscosity hypothesis

$$-\overline{u'_i u'_j} = \nu_t\left(\frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial \overline{u_j}}{\partial x_i}\right) - \frac{2}{3}\kappa\delta_{ij}$$

## II) κ–ε model

$$\frac{\mu_t}{\rho} = \nu_t = C_\mu \frac{\kappa^2}{\varepsilon}$$

$$\frac{\partial (U_i \kappa)}{\partial x_i} = \frac{\partial}{\partial x_i}\left(\frac{\nu_t}{\sigma_k}\frac{\partial \kappa}{\partial x_i}\right) - \overline{u_i u_j} S_{ij} - \varepsilon$$

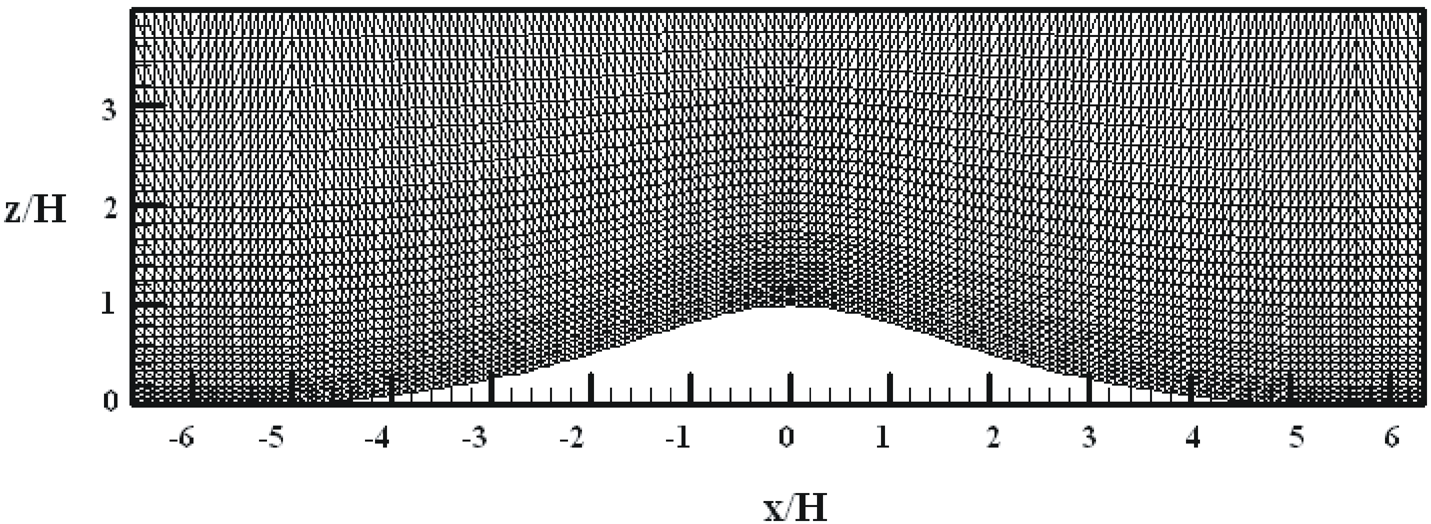
**Turbulent Kinetic Energy**

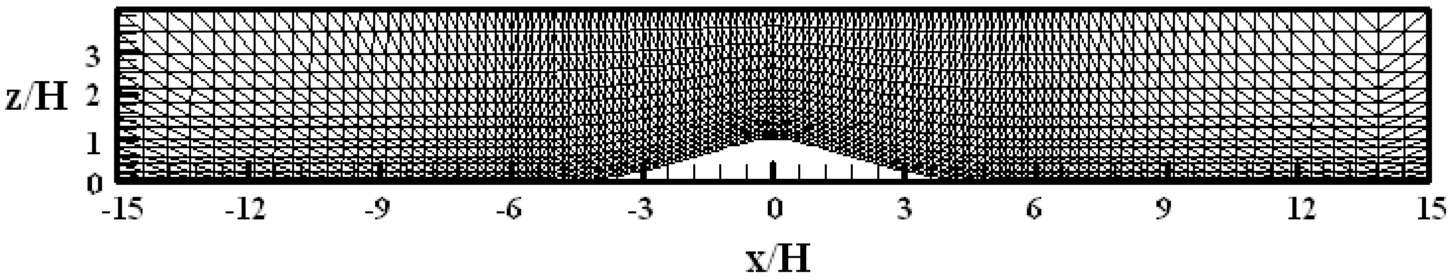
$$\frac{\partial (U_i \varepsilon)}{\partial x_i} = \frac{\partial}{\partial x_i}\left(\frac{\nu_t}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial x_i}\right) - \frac{\varepsilon}{\kappa}\left(C_1 \overline{u_i u_j} S_{ij} + C_2 \varepsilon\right)$$

**Dissipation Rate by Mass Unit**

LABORATÓRIO DE MECÂNICA DA TURBULÊNCIA

# Numerical simulation, finite elements, 2D, incompressible flow, κ–ε model



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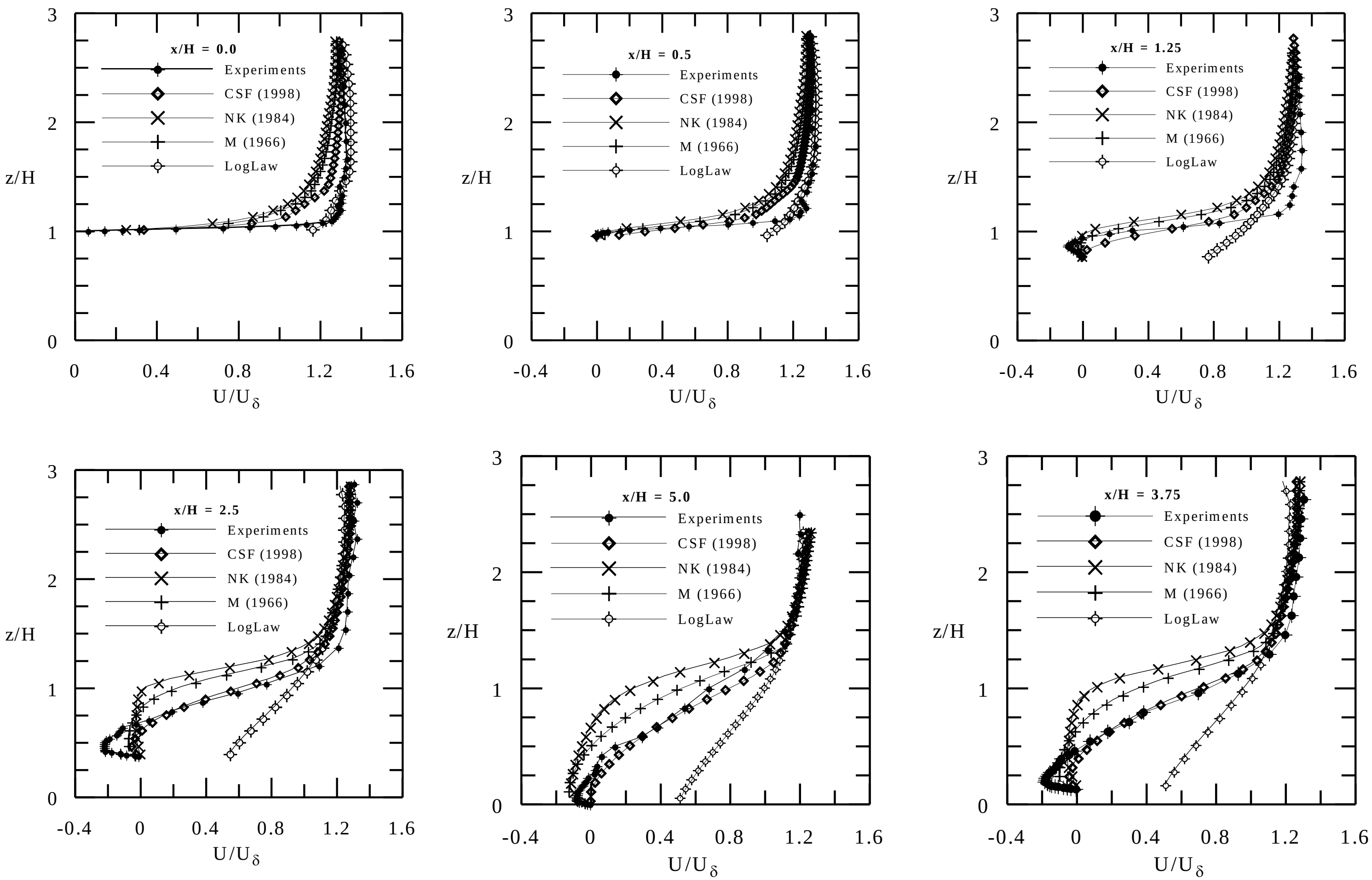
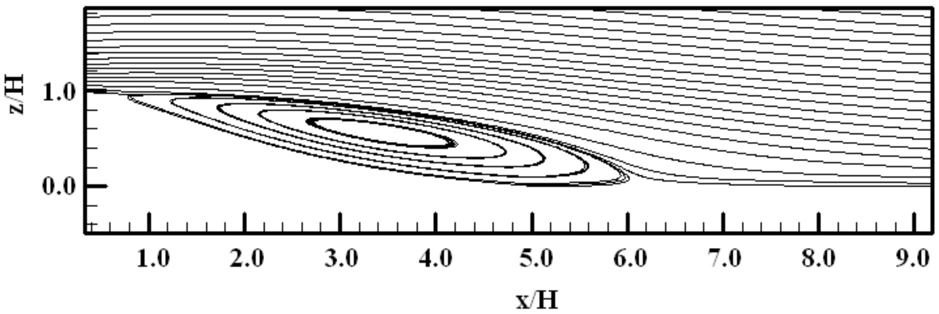
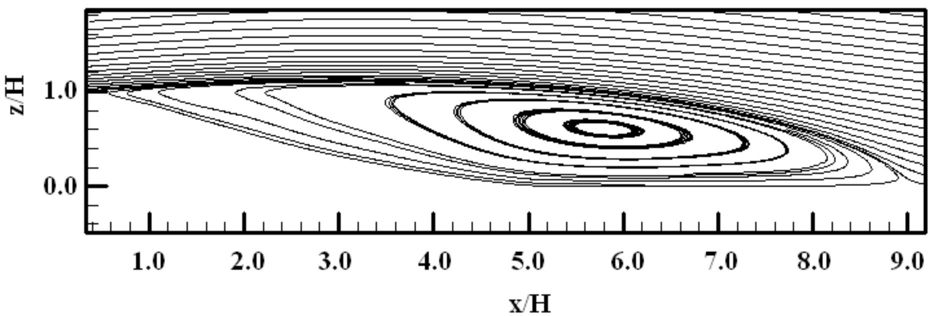
# Numerical simulation: Mean velocity



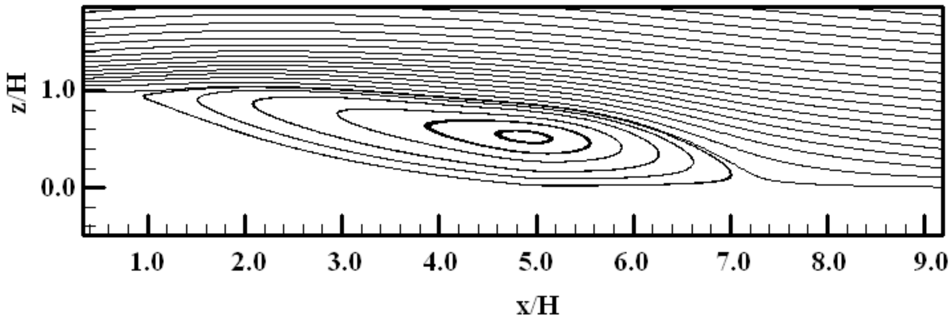
# Numerical simulation: separation region




**Cruz and Silva Freire (1998)**




**Narayama & Koyama (1984)**




**Mellor (1966)**

# Eddy Viscosity vs RSM

## RANS Equations

$$\frac{\partial \overline{u_i}}{\partial x_i} = 0$$

$$\frac{\partial \overline{u_i}}{\partial t} + u_i \frac{\partial \overline{u_j}}{\partial x_i} = -\frac{1}{\rho}\frac{\partial \overline{p}}{\partial x_i} + \frac{\partial}{\partial x_j}\left(\upsilon \frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial \overline{u_j}}{\partial x_i}\right) - \frac{\partial}{\partial x_j}\left(\overline{u'_i u'_j}\right)$$

Term to be modelled

fluctuating quantity, turbulent stresses

## I) Eddy viscosity hypothesis

$$-\overline{u_i' u_j'} = \frac{\overline{\tau_{ij}}}{\rho} = \nu_t\left(\frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial \overline{u_j}}{\partial x_i}\right) - \frac{2}{3}\kappa\delta_{ij}$$

## II) The Reynolds stress transport equation for ω based models

$$\frac{\partial \tau_{ij}}{\partial t} + \frac{\partial \overline{u}_k \tau_{ij}}{\partial x_k} = P_{ij} + \frac{2}{3}\beta' \omega\kappa\delta_{ij} - P_{ij} + \frac{\partial}{\partial x_k}\left(\left(\upsilon + \frac{\upsilon_t}{\sigma^*}\right)\frac{\partial \tau_{ij}}{\partial x_k}\right)$$

# Numerical simulation: eddy viscosity turbulence models

## Standard k-♏ model (Launder and Spalding 1974))

$$\mu_t = C_\mu \rho \frac{\kappa^2}{\varepsilon}$$

$$\frac{\partial (U_i \kappa)}{\partial x_i} = \frac{\partial}{\partial x_i}\left(\frac{\nu_t}{\sigma_k}\frac{\partial \kappa}{\partial x_i}\right) - \overline{u_i u_j} S_{ij} - \varepsilon$$

$$\frac{\partial (U_i \varepsilon)}{\partial x_i} = \frac{\partial}{\partial x_i}\left(\frac{\nu_t}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial x_i}\right) - \frac{\varepsilon}{\kappa}\left(C_1 \overline{u_i u_j} S_{ij} + C_2 \varepsilon\right)$$

## k-◆ model (Wilcox (1986) )

$$\mu_t = \rho \frac{\kappa}{\omega}$$

$$\frac{\partial \kappa}{\partial t} + \overline{u_i}\frac{\partial \kappa}{\partial x_i} = \frac{\partial}{\partial x_i}\left[\left(\upsilon + \left(\frac{\upsilon_t}{\sigma_\kappa}\right)\right)\frac{\partial \kappa}{\partial x_i}\right] + P_k - \beta' \kappa\omega$$

$$\frac{\partial \omega}{\partial t} + \overline{u_i}\frac{\partial \omega}{\partial x_i} = \frac{\partial}{\partial x_i}\left[\left(\upsilon + \left(\frac{\upsilon_t}{\sigma_\omega}\right)\right)\frac{\partial \omega}{\partial x_i}\right] + \alpha \frac{\omega}{k} P_k - \beta \omega^2$$

$$P_k = -\overline{u_i' u_j'} \frac{\partial \overline{u_j}}{\partial x_i}$$

# SST κ–ω Model

**The Shear Stress Transport (SST) κ–ω Model, accounts for turbulent shear stress transport by considering**

$$\upsilon_{\tau} = \frac{\alpha_1 \kappa}{max( \alpha_1 \omega , SF_2 )}$$

**where $F_2$ is a blending function, and S is an invariant measure of the strain rate.**

**The blending function $F_2$ is given by**

$$F_2 = tanh(arg_2^2)$$

$$arg_2 = max( \frac{2\sqrt{k}}{\beta' \omega y}, \frac{500\,\upsilon}{y^2 \omega})$$

LABORATÓRIO DE MECÂNICA DA TURBULÊNCIA

# BSL Reynolds Stress Model

$$\frac{\partial \tau_{ij}}{\partial t} + \frac{\partial \bar{u}_k \tau_{ij}}{\partial x_k} = P_{ij} + \frac{2}{3}\beta' \omega \kappa \delta_{ij} - P_{ij} + \frac{\partial}{\partial x_k}\left(\left(\upsilon + \frac{\upsilon_t}{\sigma^*}\right)\frac{\partial \tau_{ij}}{\partial x_k}\right)$$

$$\frac{\partial \omega}{\partial t} + \frac{\partial \bar{u}_k \omega}{\partial x_k} = \alpha_3 \frac{\omega}{\kappa} P_\kappa - \beta_3 \omega^2 + \frac{\partial}{\partial x_k}\left(\left(\upsilon + \frac{\upsilon_t}{\sigma_{\omega 3}}\right)\frac{\partial \omega}{\partial x_k}\right) + (1 - F_1)^2 \frac{1}{\sigma_2 \omega}\frac{\partial \kappa}{\partial x_k}\frac{\partial \omega}{\partial x_k}$$

$\sigma^* = 2$ $\quad \alpha = \frac{\beta}{\beta'} - \frac{k^2}{\sigma(\beta')^{0.5}} = 5/9$ $\quad \beta = 0.075$ $\quad k = 0.41$

**The blending coefficients are found from a linear interpolation through the relation**

$$\phi_3 = F\,\phi_1 + (1-F)\,\phi_2$$

$$F = \tanh(arg^4)$$

$$CD_{\kappa\omega} = \max\left(2\rho \frac{1}{\sigma_{\kappa-\varepsilon}\omega}\frac{\partial \kappa}{\partial x_j}\frac{\partial \omega}{\partial x_j}, 10^{-10}\right) \qquad arg = \min\left(\max\left(\frac{\sqrt{k}}{\beta' \omega y}, \frac{500\,\upsilon}{y^2 \omega}\right), \frac{4\rho\kappa}{CD_{\kappa\omega}\sigma_{k-\varepsilon} y^2}\right)$$

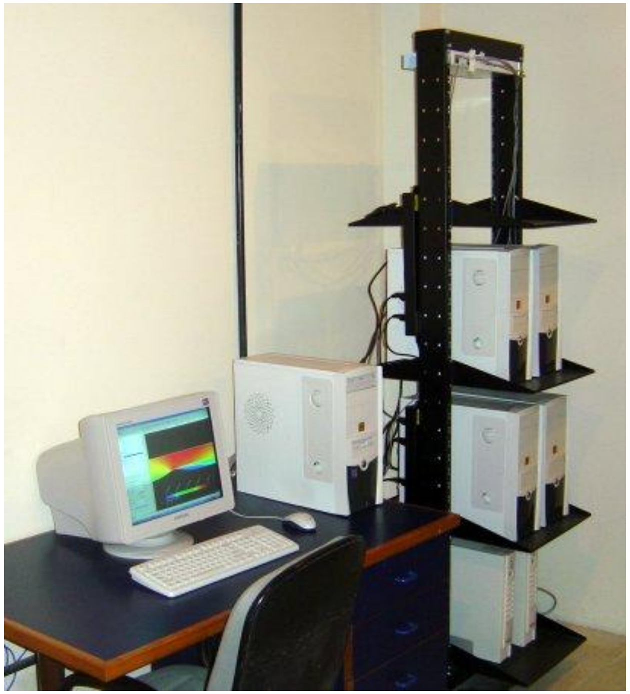
# Cluster Configuration

- Front End:
  - Intel D875PBZ Motherboard (With on-board Gigabit Ethernet network interface)
  - Pentium 4, 3.0Gz, 1Mb Cache
  - 1 Gb DDR400 in dual mode (2 x 512 Mb)
  - 200 GB SATA HD

- Nodes (4):
  - Intel D875PBZ Motherboard (With on-board Gigabit Ethernet network interface)
  - Pentium 4, 3.0Gz, 1Mb Cache
  - 1 Gb DDR400 in dual mode (2 x 512 Mb)
  - 40 GB ATA HD

- 3COM Gigabit Ethernet Switch 3C16478

- 2 “APC Back-UPS RS 1500” 1500 VA UPS

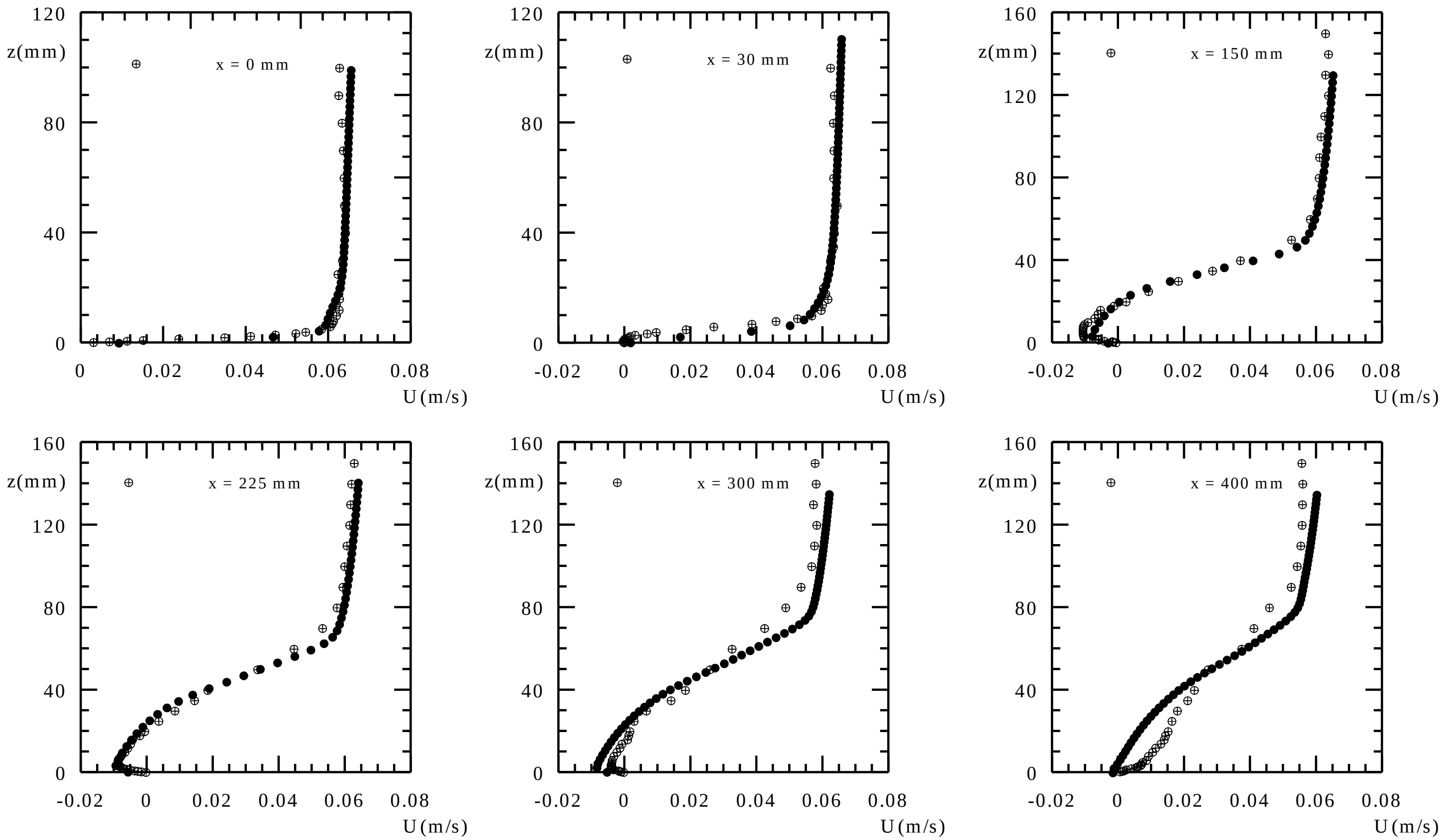


LABORATÓRIO DE MECÂNICA DA TURBULÊNCIA

# Simulation Details

- SST (Eddy Viscosity)
  - 400,000 Elements (Hexaedra)
  - Inlet speed of 0.0482 m/s
  - Smooth wall
  - Total run time 5:09:32
- BSL (Reynolds Average)
  - 350,000 Elements (Hexaedra)
  - Inlet speed of $0{,}0482\left(\frac{z}{\delta}\right)^{0{,}142857}$ m/s
  - Smooth wall
  - Total run time 5:23:02
- SSG (Reynolds Average)
  - 350,000 Elements (Hexaedra)
  - Inlet speed of $0{,}0482\left(\frac{z}{\delta}\right)^{0{,}142857}$ m/s
  - Smooth wall
  - Total run time 2:23:40

# Longitudinal velocity profiles in the recirculation region, k-◆ based SST model



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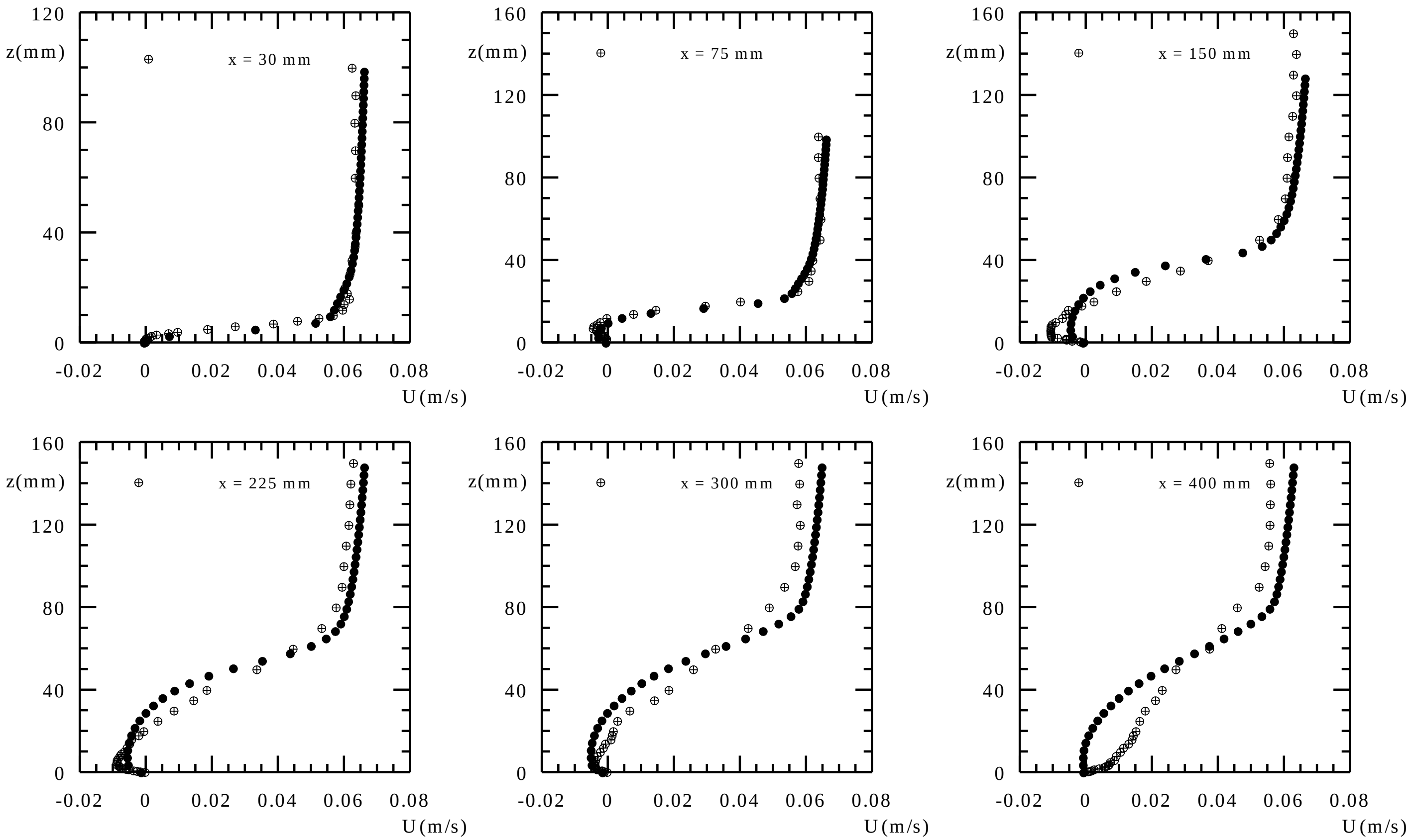
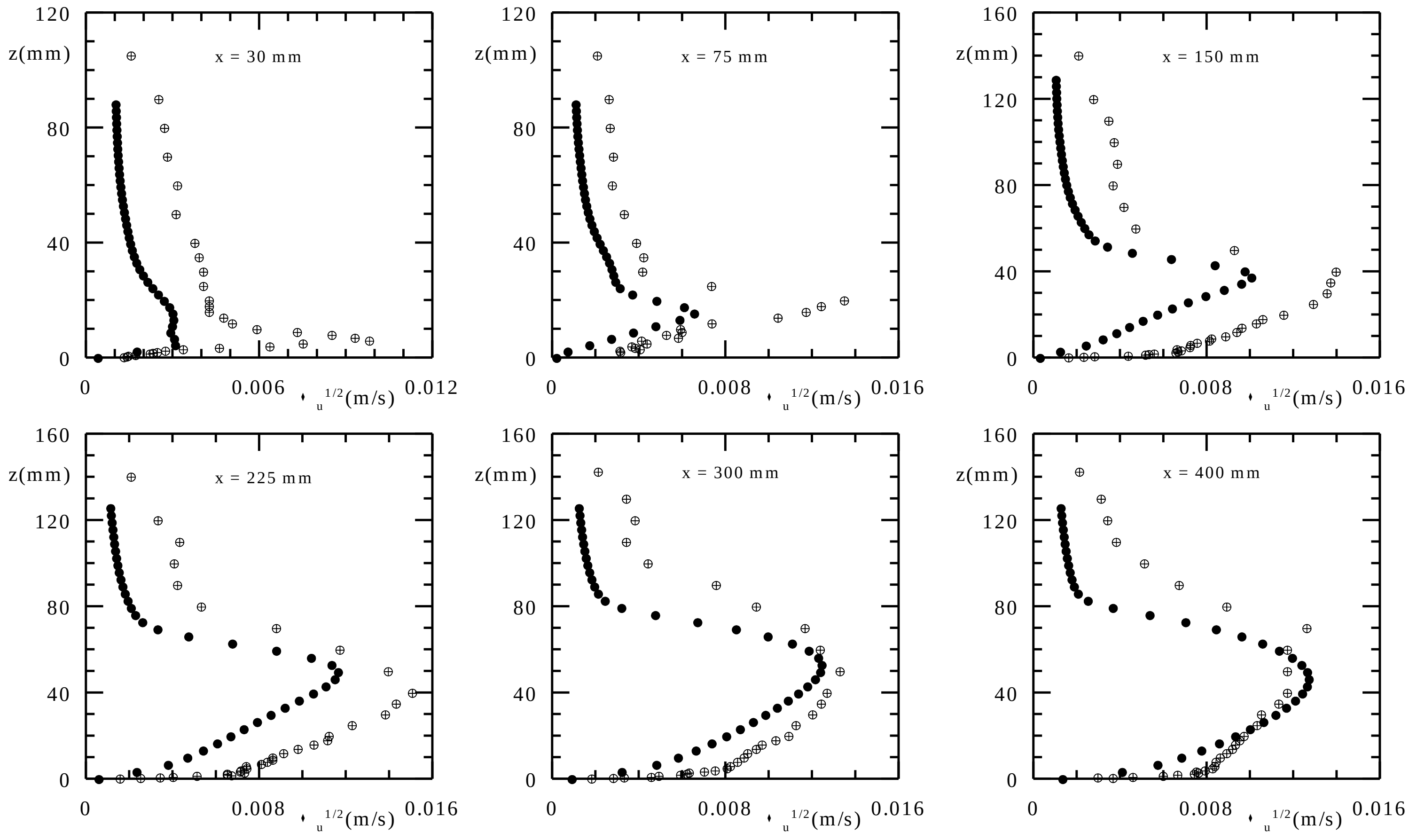
# Longitudinal velocity profiles in the recirculation region, BSL Reynolds stress model



LABORATÓRIO DE MECÂNICA DA TURBULÊNCIA

# Longitudinal turbulent velocity profiles in the recirculation region, BSL Reynolds stress model



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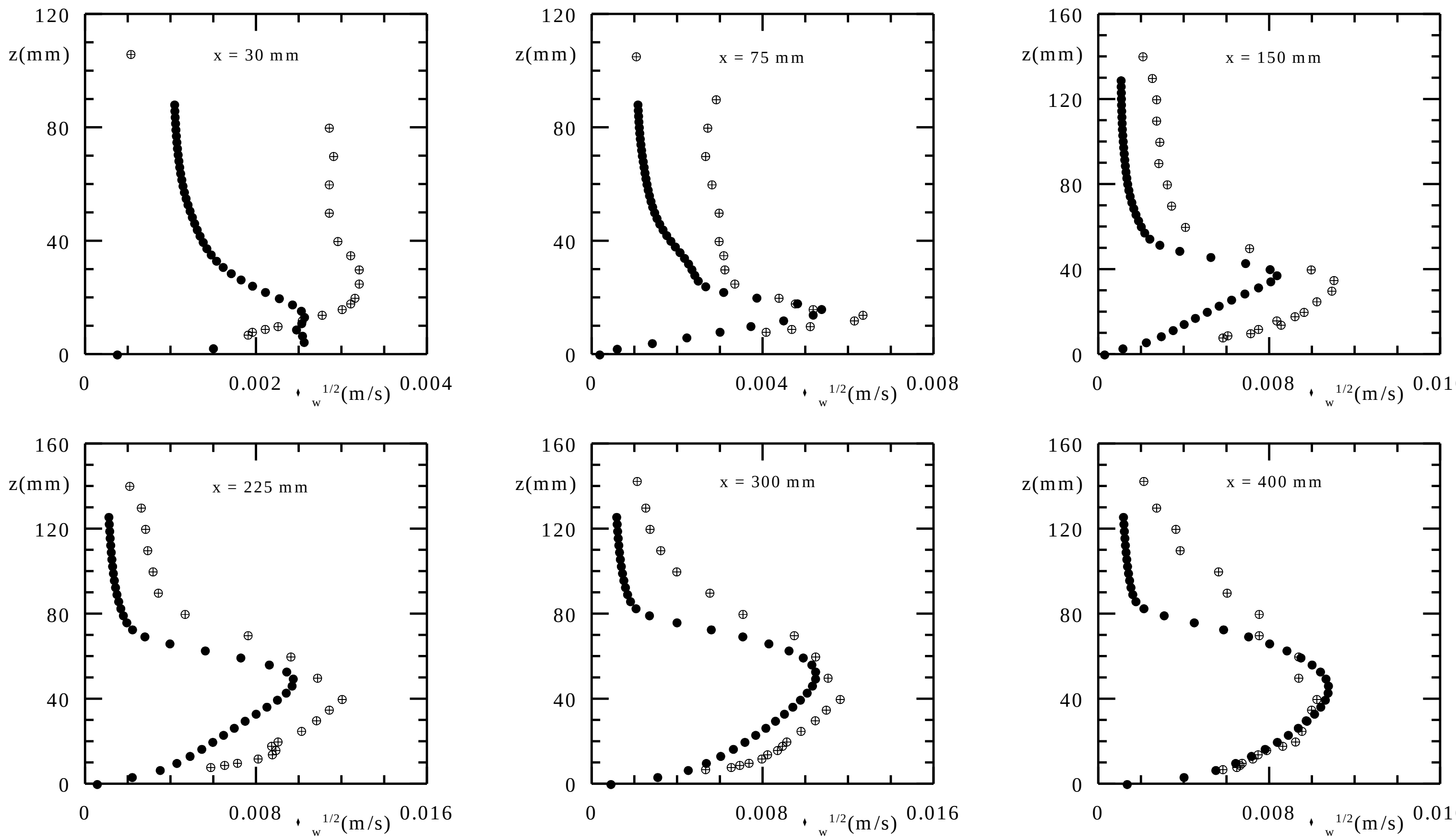
# Vertical turbulent velocity profiles in the recirculation region, BSL Reynolds stress model



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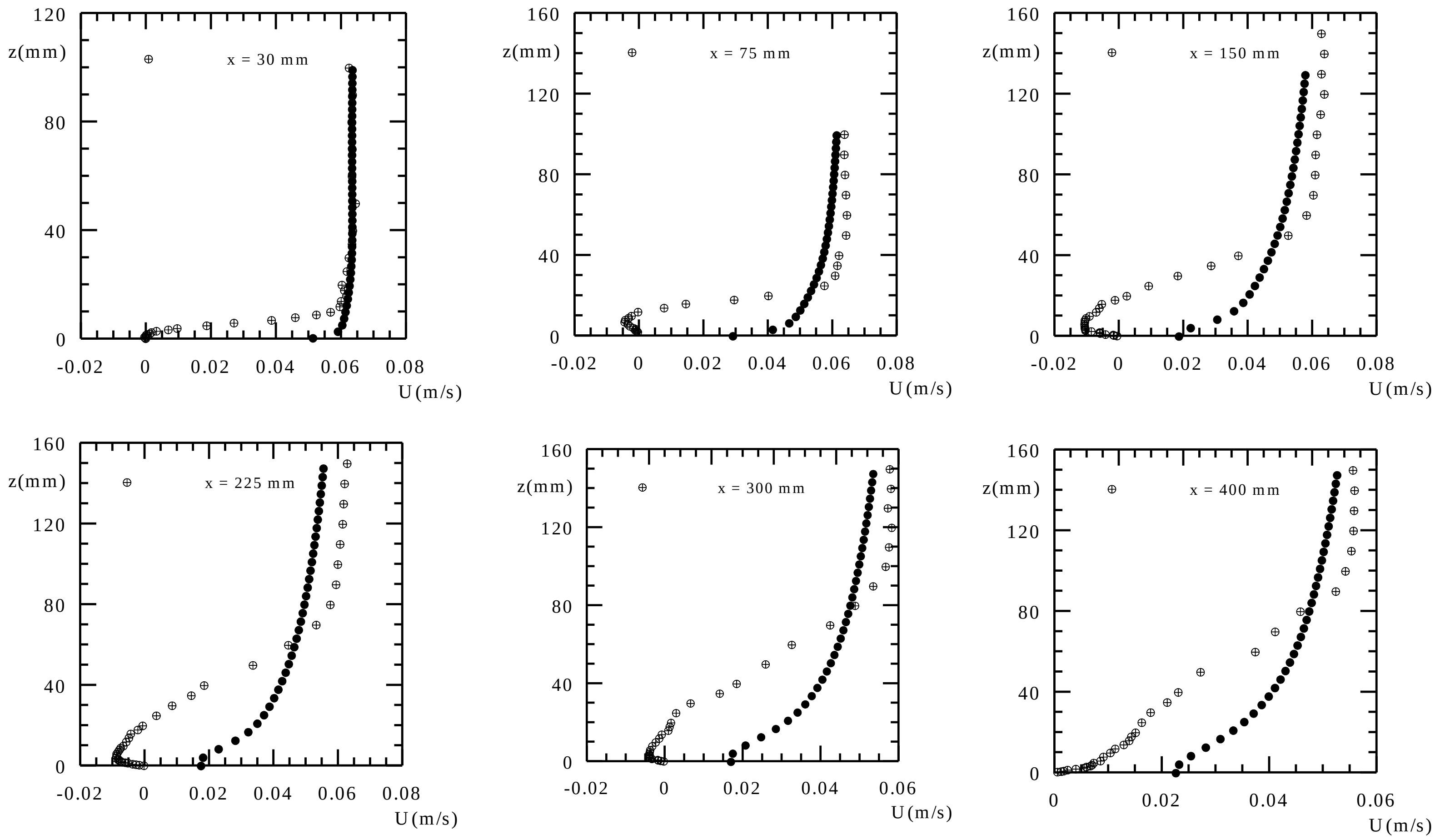
# Longitudinal velocity profiles in the recirculation region, SSG model.



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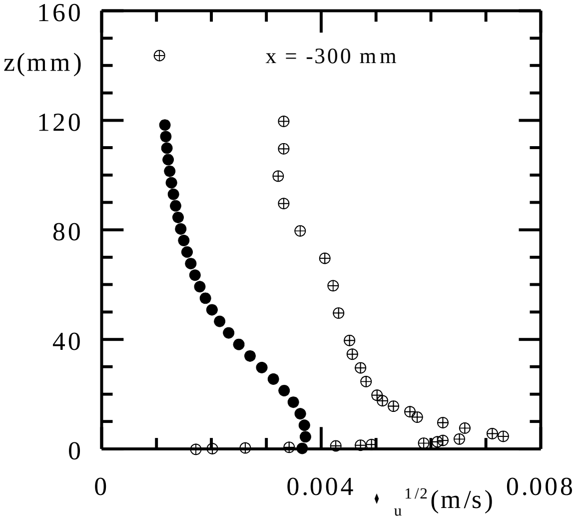
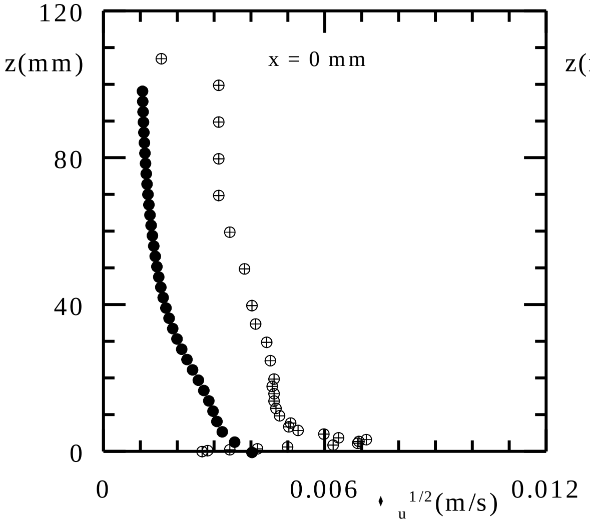
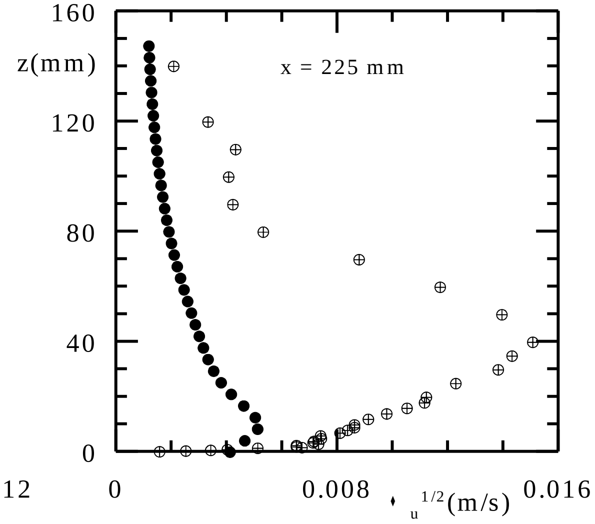
# Representative turbulence velocity profiles, SSG Model



# Final Remarks

1. Present work has shown how different law of the wall formulations can be used to describe the flow over a step hill.

2. Validation of theory was provided by H-W and LDA measurements over a model hill.

3. Further numerical simulations of the problem were presented through two-equation differential models and RSM.

4. Future work will focus on 3d geometries and on the description of the temperature field.

LABORATÓRIO DE MECÂNICA DA TURBULÊNCIA

# Acknowledgements

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